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THESIS

AN EXPERIMENTAL STUDY OF ELECTROMAGNETIC LORENTZ FORCE AND RAIL RECOIL

by

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December 2009

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13. ABSTRACT (maximum 200 words)

Understanding whether recoil forces are seated in the rails of any electromagnetic launch technology, including railguns, is critical for efficient development and design. Several theoretical and experimental researchers have produced multiple published papers characterizing rail recoil. These papers are not definitive and often conflict. An experiment has been developed that allows for the simultaneous measurements of the quasi-static Lorentz force on the armature and rail recoil. The primary challenge in quantifying these forces is removing the mechanical coupling required to construct the necessary circuit while maintaining electrical connectivity. Liquid metal Ga/In eutectic was used to conduct electricity while mechanically decoupling the rails from the rest of the circuit. Force measurements show that the force on the armature increases as the square of the current while the indicated reaction force on the rails is an artifact of the experiment. These recoil forces measured <1% of the force on the armature. We conclude that the recoil, or corresponding equal and opposite reaction force to the force on the armature, is not seated in the rails.

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AN EXPERIMENTAL STUDY OF ELECTROMAGNETIC LORENTZ FORCE AND RAIL RECOIL

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ABSTRACT

Understanding whether recoil forces are seated in the rails of any electromagnetic launch technology, including railguns, is critical for efficient development and design. Several theoretical and experimental researchers have produced multiple published papers characterizing rail recoil. These papers are not definitive and often conflict. An experiment has been developed that allows for the simultaneous measurements of the quasi-static Lorentz force on the armature and rail recoil. The primary challenge in quantifying these forces is in removing the mechanical coupling required to construct the necessary circuit while maintaining electrical connectivity. Liquid metal Ga/In eutectic was used to conduct electricity while mechanically decoupling the rails from the rest of the circuit. Force measurements show that the force on the armature increases as the square of the current while the indicated reaction force on the rails is an artifact of the experiment. These recoil forces measured <1% of the force on the armature. We conclude that the recoil, or corresponding equal and opposite reaction force to the force on the armature, is not seated in the rails.

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I. INTRODUCTION

A. MOTIVATION

For over 200 years, electromagnetic forces have been extensively researched. During 1802, Gian Domenico Romagnosi noticed that a magnetic needle deflected when electricity from a crude battery was turned on and off [1]. Less than 20 years later, Hans Christian Oersted independently discovered the same phenomenon, and through further experiments, deduced that a current carrying wire produces a magnetic field [2]. This electric force was put to use in the first electric gun by Joachim Hansler in 1844 [3], some 48 years before Lorentz introduced his force equation in 1892 [4].

Even though the Lorentz force has been known for well over 100 years, its corresponding reaction force is still a topic of controversy. Numerous theoretical and experimental researchers have tackled this issue, with a wide variety of results [5]-[27]. An experiment by Graneau [28], led him to conclude that there are longitudinal recoil forces seated in the rails. Witalis [26], asserts that relativistic recoil forces are exerted on the rails in a direction parallel to the rails [26].

Allen and Jones [5,6] state that Graneau is incorrect. They claim railgun <u>rails</u> will not recoil, but instead recoil occurs at the <u>breech</u> due to reflected waves, which create "electric pressure" via "electromagnetic momentum." Also, Marshall and Woods [16] rebut Witalis's work by

combining theory with empirical observations from the Canberra railgun. They conclude recoil forces are not seated in the rails.

Sadedin suggests that momentum can be conserved in railguns by modeling recoil forces as a gas pressure [22]. Graneau refutes this notion by stating that the Lorentz force law fails to predict where recoil is seated [18]. Clearly there is room for experiment to resolve this controversy.

B.   OBJECTIVE

The focus of this thesis was to determine if electromagnetic recoil forces are seated in the rails. Experimental research was conducted to produce quantitative evidence that will definitively answer this question. It should be noted that the scope of this thesis does not include determining where else recoil forces may be seated. Specifically, this experiment quasi-statically measures the force that accelerates the armature and compares that with the measured recoil force.

C.   BACKGROUND

Railguns operate through the interaction of flowing electrons in the armature with the magnetic fields produced by electric current in the rails. This interaction produces what is called the Lorentz force,

$$F = \frac{1}{2}L'I^2 \tag{1}$$

which is exerted on the armature and accelerates it down the barrel. In this equation, L' is the inductance gradient per unit length of the rail pair, and I is the

current flowing through the rails and armature. Equation (1) is widely accepted as the force on railgun armatures [29]. Figure 1 illustrates how the Lorentz force accelerates an armature.
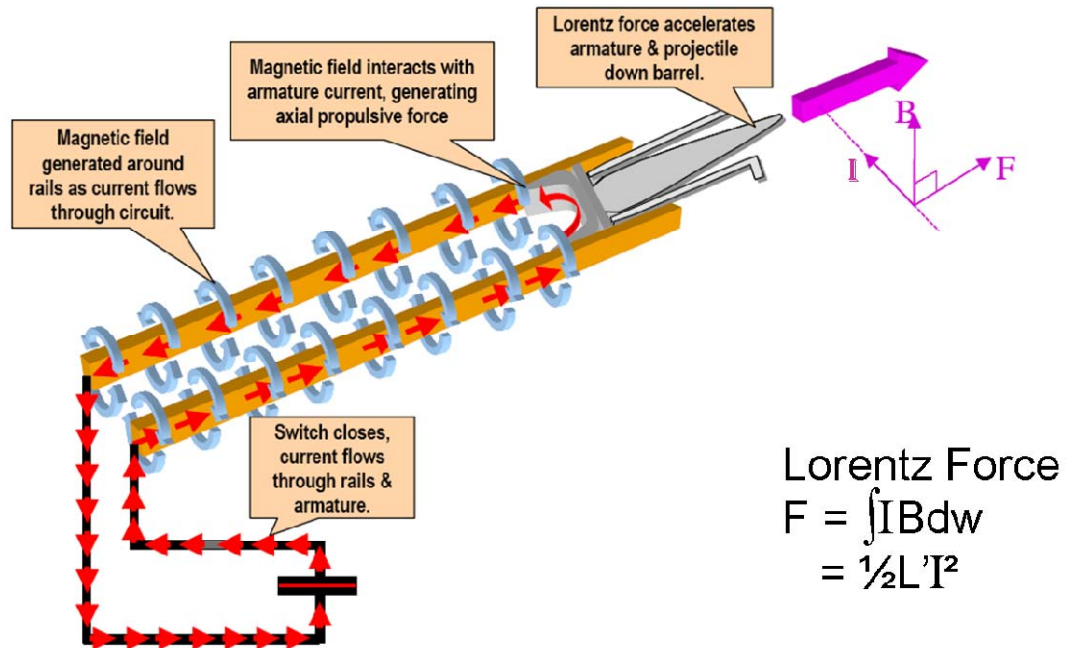

Figure 1.   Schematic illustration of railgun operation (From [29])

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II. EXPERIMENTAL SETUP

A.   PREVIOUS RESEARCH

This thesis is a continuation of LT Matthew Schroeder's research [30]. His work included the design and construction of the experimental apparatus used to conduct the research in this thesis. New modifications will be specifically mentioned during the overview of the complete experimental setup.

B.   COMPONENTS

1.   The Rails

Figure 2 shows a picture of the setup taken from the muzzle end. Fabricated from copper bar stock, the rails are approximately 3 cm wide by 0.5 cm tall. The separation between the rails is about 5 cm and they are 2 m long.
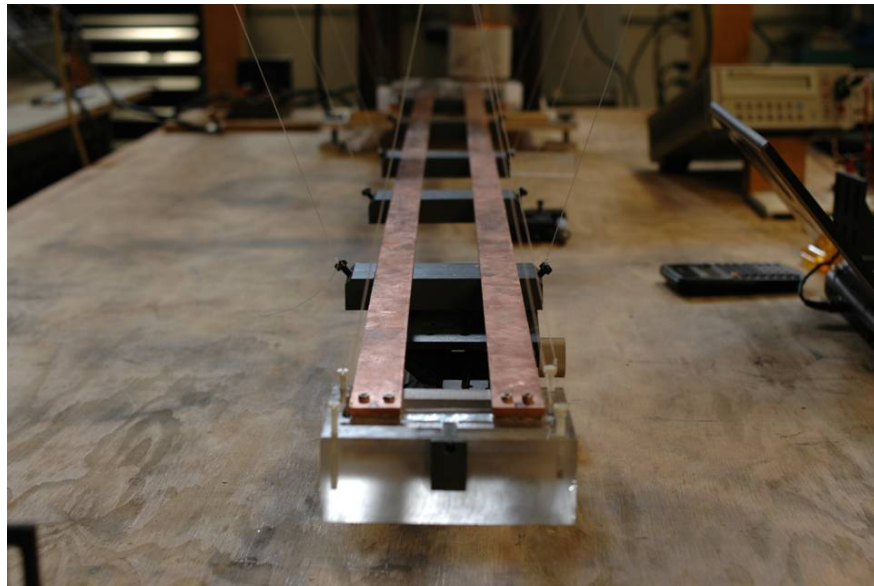

Figure 2.   Copper rails and armature

2. Pendulum Suspension

Five polyvinylchloride (PVC) blocks supported the rails. These blocks were suspended from monofilament line forming a 'V' shaped pendulum. The line was attached to two parallel 8 ft long 2 X 4 in wood beams. Figure 3 displays the suspended rails, which are free to move along the longitudinal axis of the rails. The design dimensions are given in Figure 3. The two top beams that the pendulum lines hang from are 8 ft long 2 X 4 in wood boards. The distance between the two top beams is 4 ft.
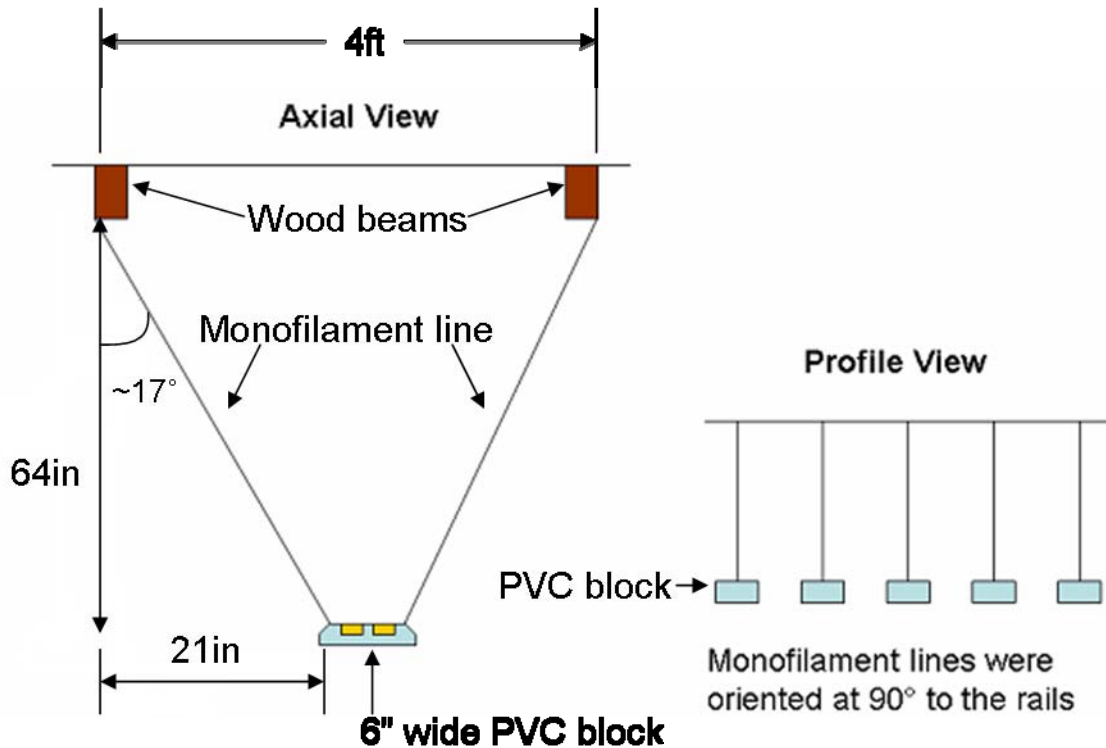

Figure 3.   Pendulum suspension system-not to scale,
(From [30])

3. Armature and Eutectic

The armature consisted of a suspended plastic block with liquid metal Gallium/Indium eutectic in a 2 cm deep polycarbonate reservoir. Copper tabs, which measured 1.8 cm deep, were attached to the rails and dipped into the eutectic. Shown in Figure 4, this interface removed most of the mechanical coupling between the rails and the armature while still allowing current flow. The inability of the fluid to sustain a shear force allowed the rail and armature to move independently of one another. The armature was suspended from four corners by monofilament line which connected to a swivel 12 in above. The swivel was connected by a single line to the three dimensional translation system pictured in Figure 5. These optical mounts contained micrometer adjustments, which provided for precise positioning of the armature in relation to the rails. Proper adjustment ensured no physical contact between the rails and the armature block.
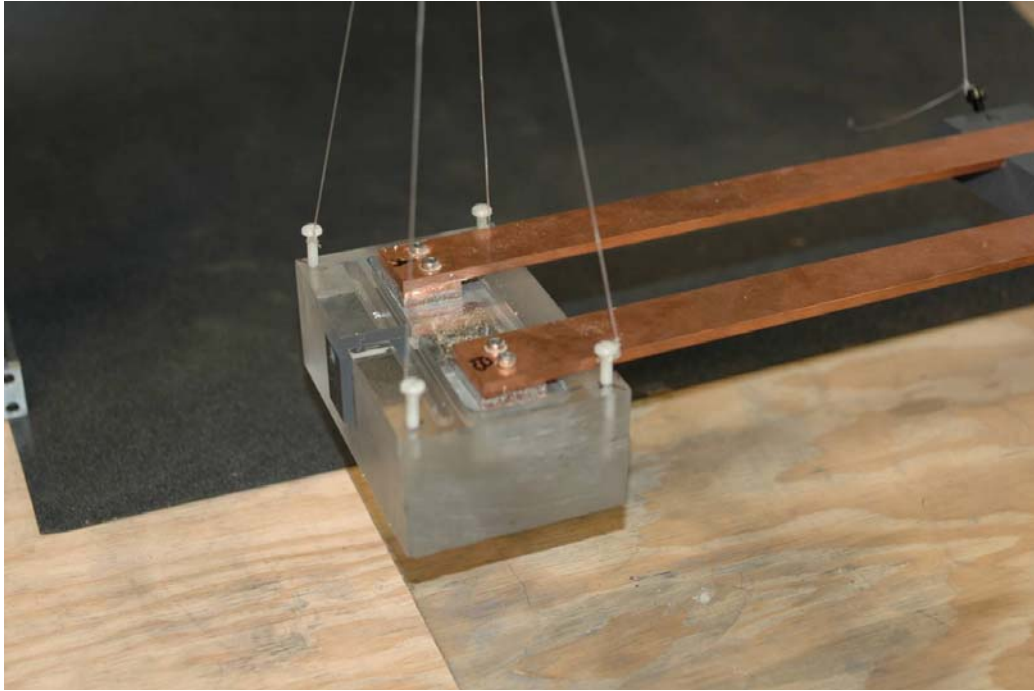
Figure 4.   Rail/armature interface

Figure 5.   3D translation system

4.   Power Supply, Switching, and Resistance

The power supply consisted of a large variable resistor in series with four Autolite 96 Platinum car batteries connected in parallel as shown in the schematic of Figure 6.   Higher currents were obtained by connecting four more batteries in parallel, thereby lowering the combined internal resistance.   Currents between 800 A and 2.7 kA were used.

The high currents required a variable resistor which was capable of dissipating the corresponding I^2R losses. The led to the use of a large stack of graphite plates as a variable resistor.   The number of plates and the compression on them could be changed to control how much current flowed through the rails.   The graphite plates were 0.5 cm thick and there were 100 plates total.   Two copper plates were moved to alter the number of graphite plates in the current path.   Small partial turns on the compression wheel adjusted the resistance by micro-ohms.   Figure 7 shows the compressible graphite plates and the two copper plates.
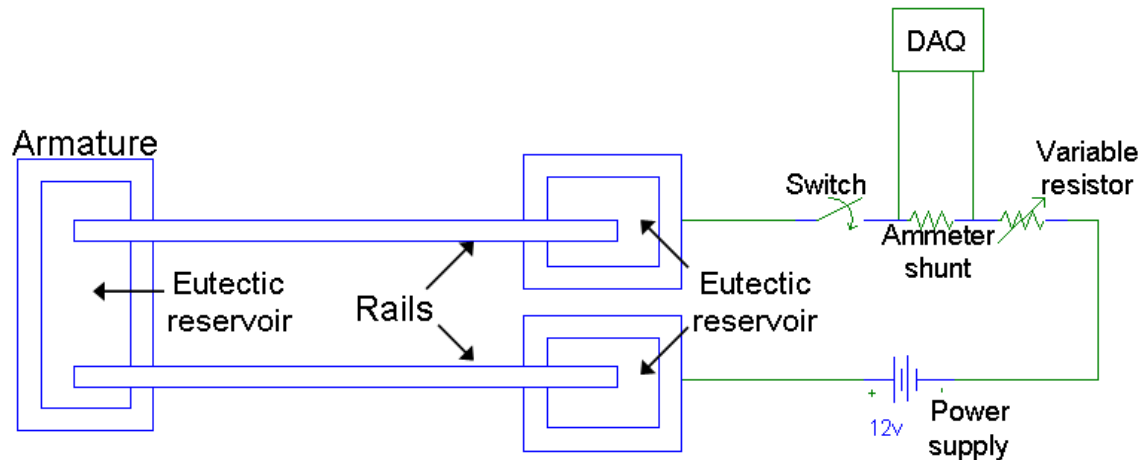

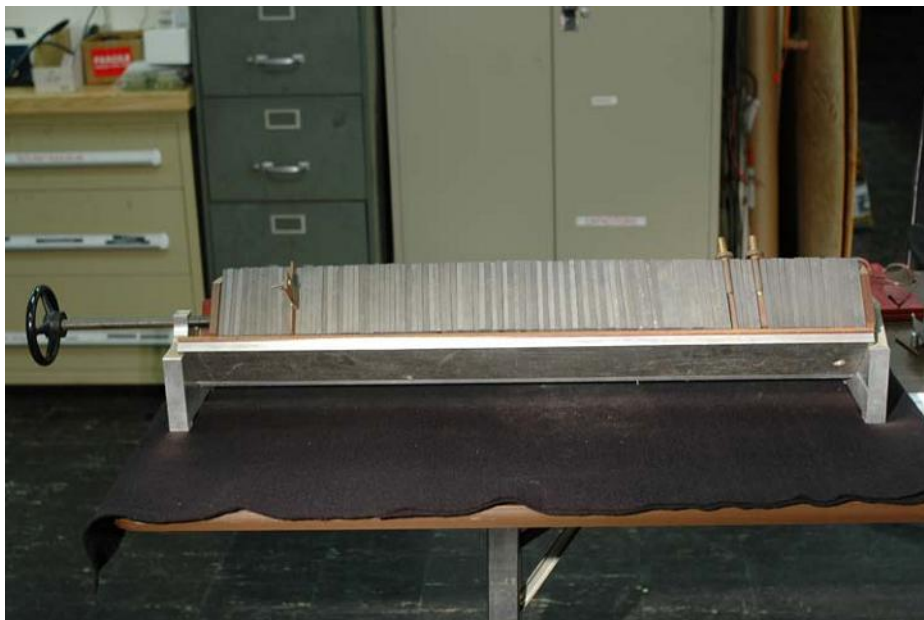Figure 6.   Schematic of model railgun circuit (not to scale)

Figure 7.   Graphite plate resistor

Current was turned on and off using the high current industrial switch shown in Figure 8. The switch was vacuum sealed and pneumatically actuated. A toggle switch was wired to control flow of an inert gas to the actuator. The actuator took approximately 1 s to close the switch, but opened in a small fraction of a second.
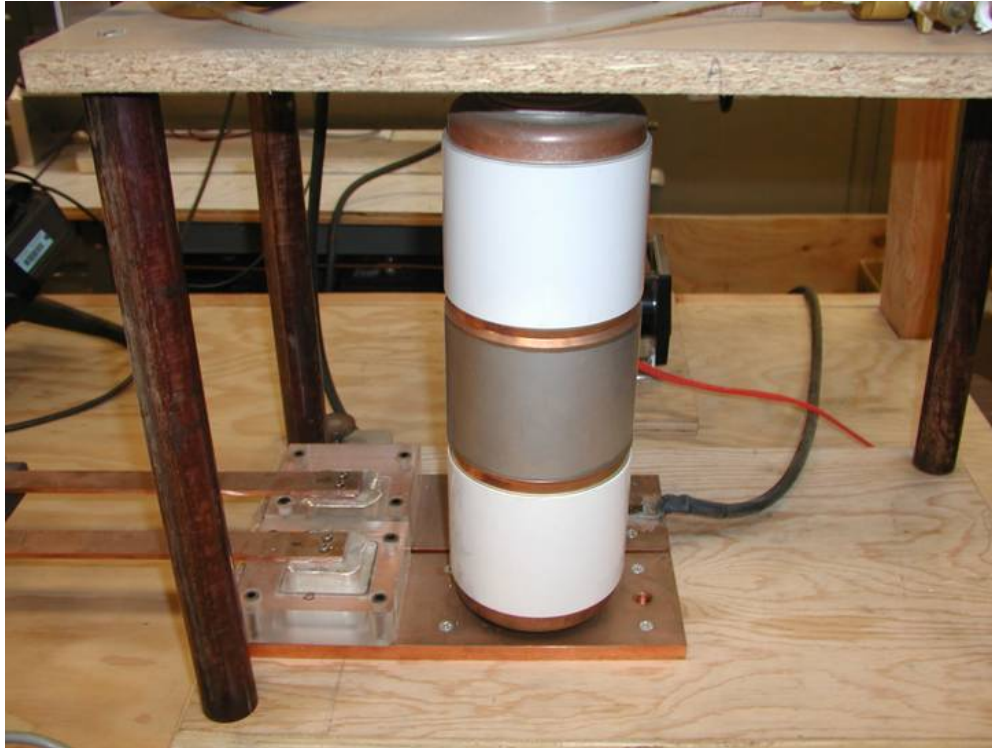
Figure 8.   Meidensha 50 kA vacuum interrupter and breech
electrical connection

Liquid metal eutectic electrically coupled the rails
to the bus bars, as shown in Figure 8.   The Gallium-Indium
eutectic had relatively small viscosity, but was highly
conductive.   Since the eutectic was unable to sustain a
shear force and the rails were suspended, the rails were
entirely free to move.   The pendulum suspension system did,
however, provide a small restoring force measured at
approximately 0.025 N.

5.   Splitting the Rails

After initial testing was complete, it was deemed
necessary to split the rails (explained in Chapter III,
section C). The rails were cut in the middle at 1 m, and then
each new end had copper tabs attached, just as at the breech

and muzzle ends.   A polycarbonate block had two reservoirs filled with eutectic.   The block was raised using a lab jack until the tabs were sufficiently submerged to complete the circuit's electrical connectivity.   Figures 9 and 10 show different views of the split rails, ready to energize.
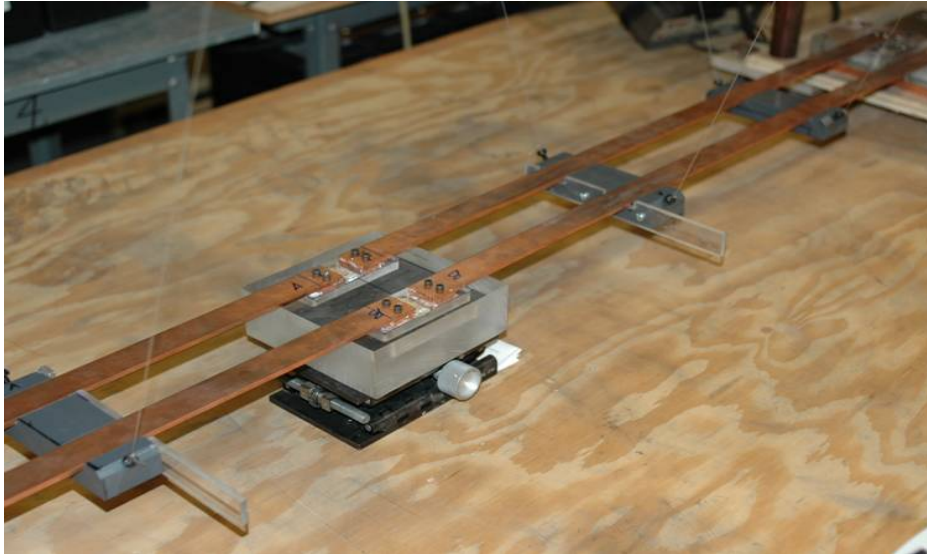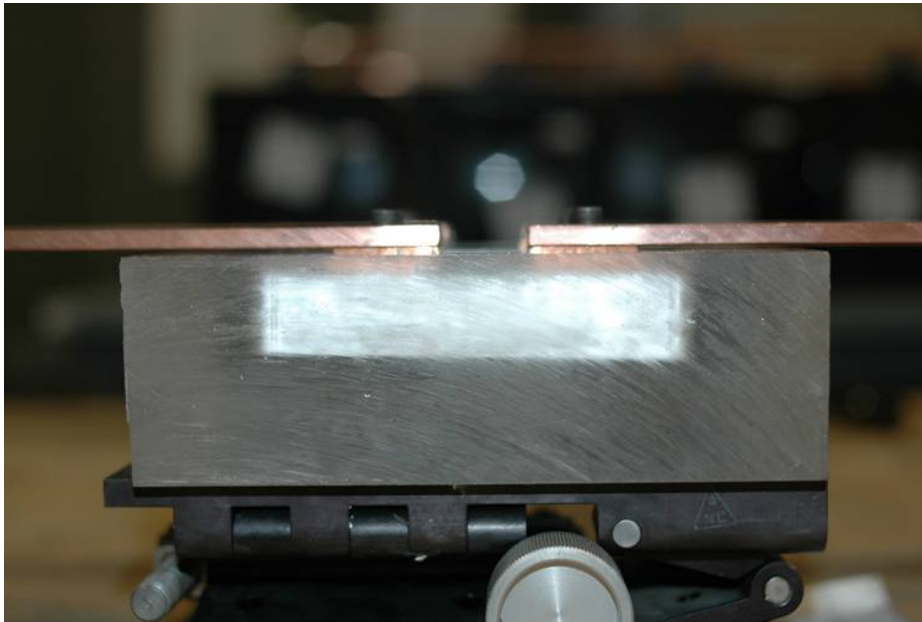

Figure 9.   Split rails

Figure 10.   Rail tabs submerged in eutectic

6.   Instrumentation

Measurement of forces was accomplished with strain gauges.   The LC305-25, by Omega Engineering, is a 2 in diameter miniature stainless steel compression load cell, shown in Figure 11.   These gauges were fixed to optical mounts, which were fastened to the table.   The micrometer slide provided precise positioning.   Each LC305-25 required a 10 v power source to operate, and produced 193 µv per Newton of force.   The deflection of the load cell for the magnitude of forces being measured was less than 0.001 in or 25 µm.
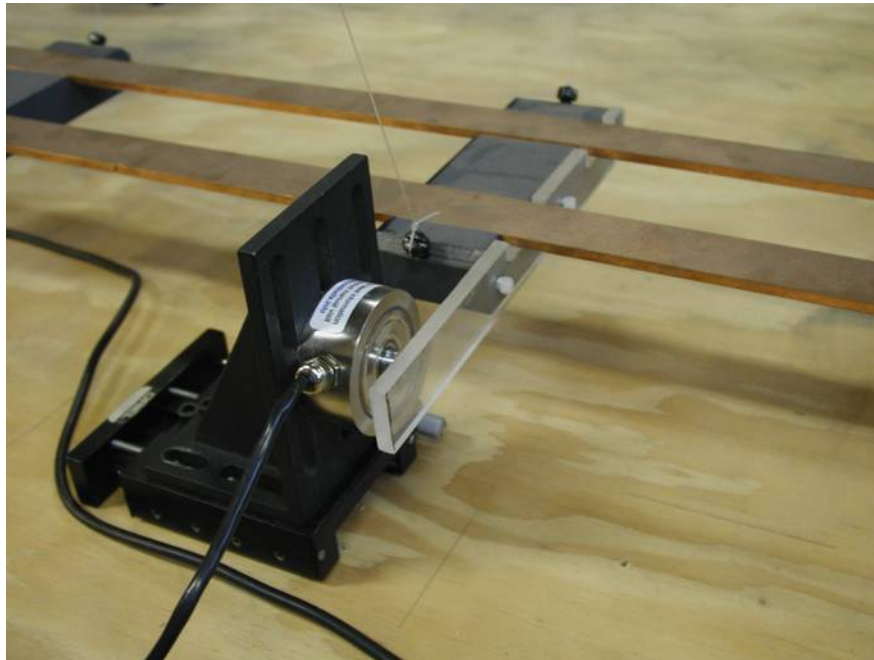

Figure 11.   LC305-25 load cell

Current through the rails was determined by use of an ammeter shunt. The shunt has a known resistance (62.5 µΩ), and the voltage drop across the shunt was input to the data acquisition converter (DAQ) for analog-to-digital conversion.   This data was then sent via USB to a PC for

Labview to process and display continuous real-time current readings. (Schroeder's research used an analog voltmeter and calculations to find the current.) Figure 12 shows the ammeter shunt with leads.
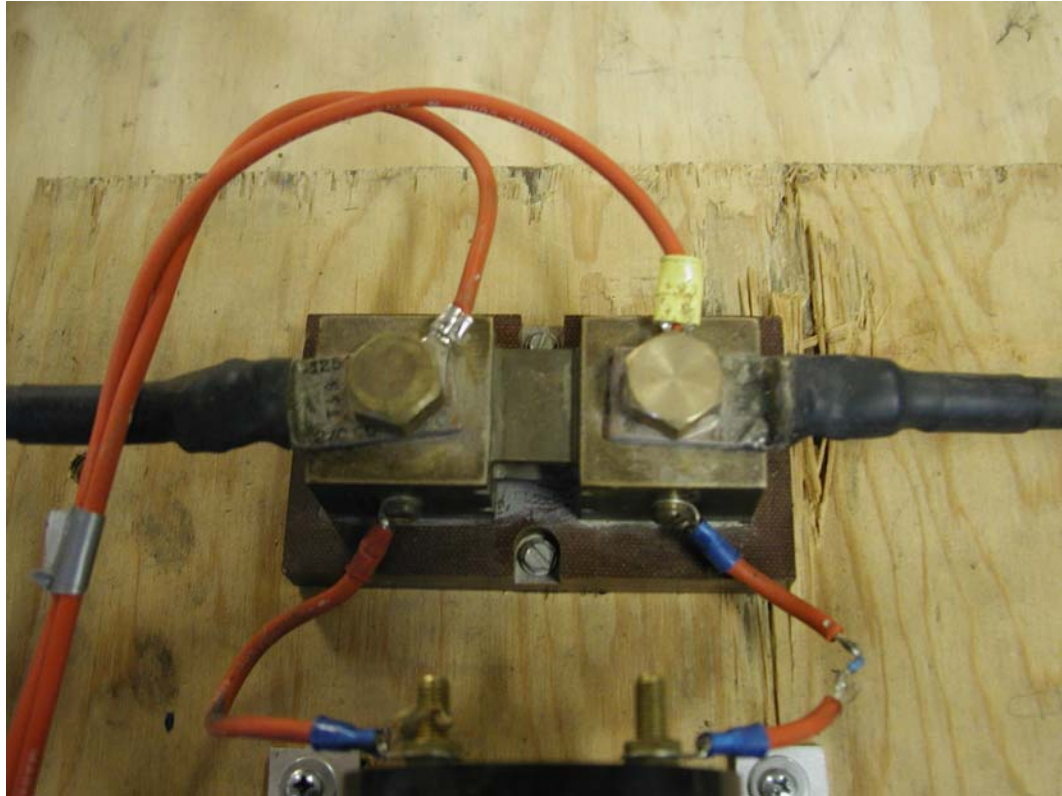

Figure 12.   Ammeter shunt with leads routed to USB-6211 DAQ

During previous research [7], the meter of choice was an Omega Engineering DP41-B-4R-A-EI 1/8 DIN ultra-high performance meter, which provided peak force measurements. The meter used in this research was the superior National Instruments USB-6211 DAQ. The 6211 provided real-time continuous data collection via 16 analog inputs with 16 bits of resolution at a sample rate of up to 250 kS/s. The USB-6211, shown in Figure 13, connects via USB to a laptop, and is utilized with a Labview program.
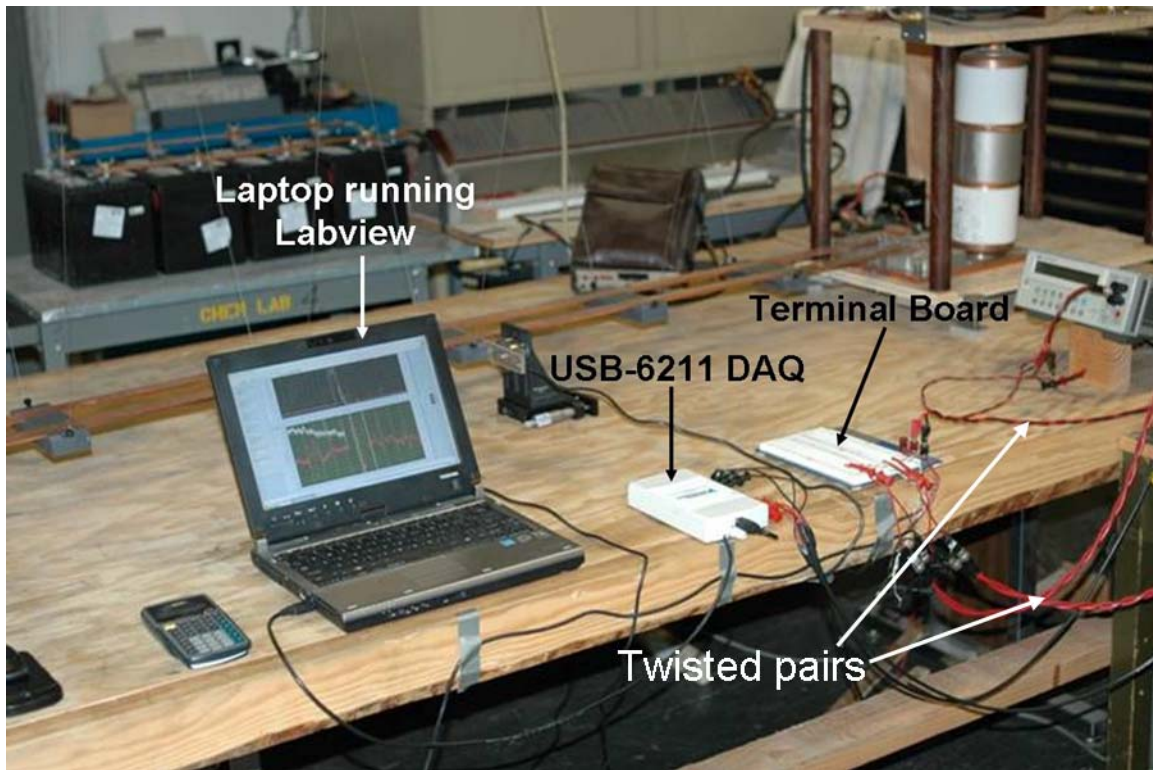
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Figure 13.   NI USB-6211 DAQ connected to laptop

The use of two Stanford Research Systems model SR560 Low Noise Preamplifiers were needed since forces on the order of 0.01 N produced voltage signals of approximately 2 µv.   These preamps provided noise filtering and amplification prior to input into the USB-6211 DAQ.   A differential input connection was required with twisted pair wire routing to minimize noise and interference, as shown in Figure 14.   The functional flow of all instrumentation is displayed in Figure 15.
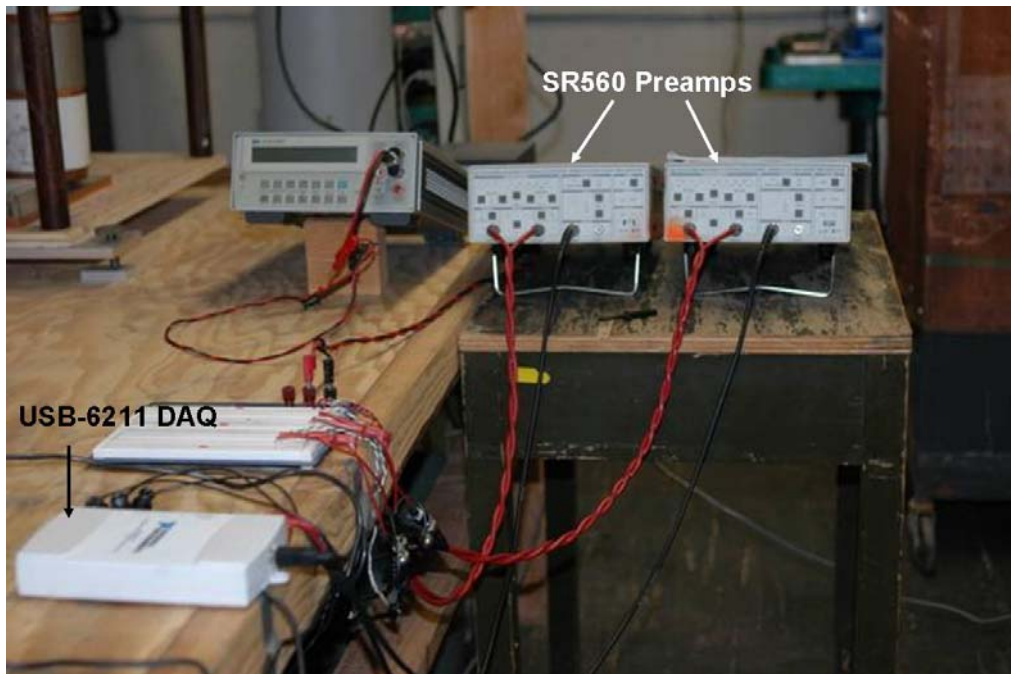
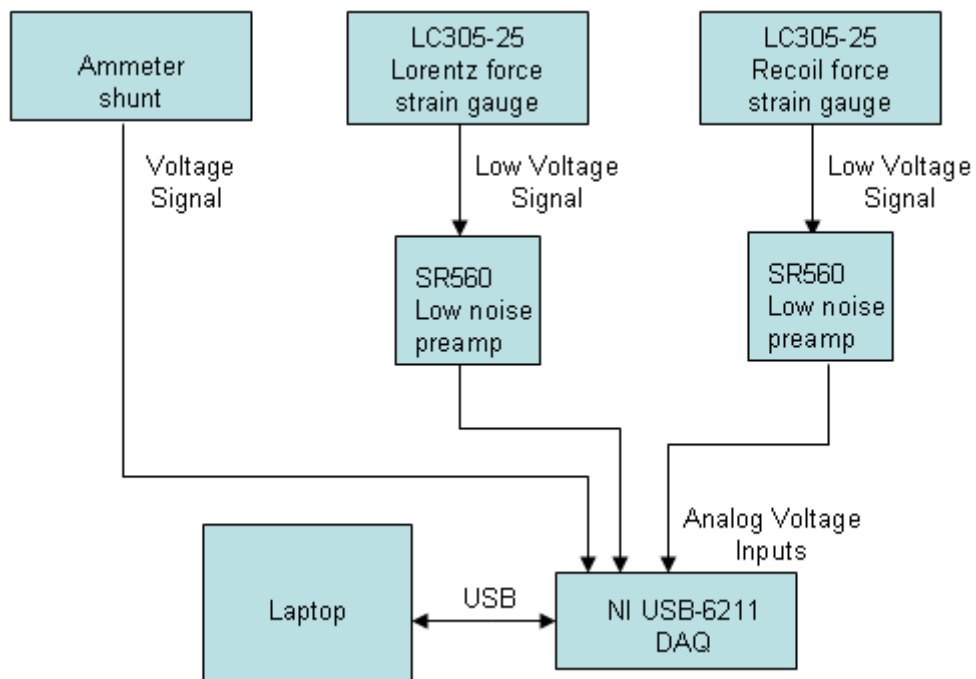Figure 14.   SR560 low noise preamplifiers connected
to USB-6211

Figure 15.   Instrumentation flow chart

III. EXPERIMENTAL PROCEDURE

A.   SCOPE

The objective of this experiment was to determine if recoil forces were seated in the rails. This was attacked in two phases: 1) simultaneous measurement of the Lorentz force and rail recoil, and 2) splitting the rails to further examine the possibility of forces seated in the rails.

B.   RECORDING LORENTZ FORCE AND RECOIL

To capture these forces, one load cell was mounted in front of the armature, as shown in Figure 16(a). A second load cell was positioned to detect recoil forces as shown in Figure 16(b).

Figure 16.   Lorentz force armature load cell (a),
recoil force load cell (b)

The circuit was energized by activating the toggle switch, which initiated the closing of the vacuum interrupter.   After approximately 4 seconds, the circuit was de-energized.   Data recording was initiated approximately 3 seconds before the circuit was energized, and ran for 10 seconds.

C. SPLITTING THE RAILS

After the rails failed to recoil at currents as high as 2.6 kA and Lorentz forces above 1.5 N, the rails were split to investigate for internal stress. It was not believed, but thought possible, that there might be a force on the rails from the breech, which could cancel recoil forces. Splitting the rails properly would show if these forces existed.

After the rails were split, simultaneous force measurements were taken from the armature and the adjacent muzzle half of the rails, for different current levels. The same procedure previously mentioned for recording data was utilized. To determine if the split rails pushed toward each other, two load cells were placed accordingly, as shown in Figure 17. However, the rails were instead discovered to push apart with a small force as discussed in the Chapter IV. The load cells were repositioned accordingly to capture this force.
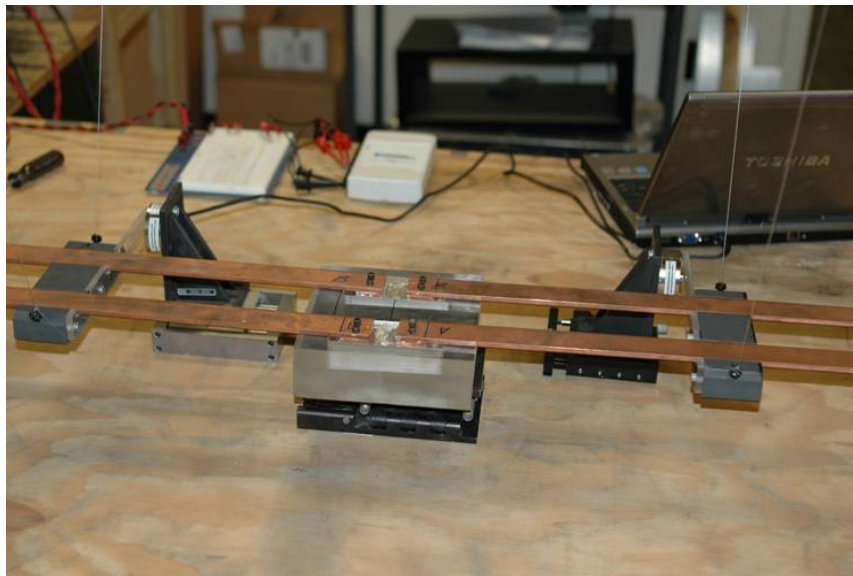

Figure 17. Load cells set up for split rails

IV. EXPERIMENTAL RESULTS

A. SIMULTANEOUS LORENTZ AND RECOIL FORCES

Force measurements were recorded for current levels up to 2700 amps. Figure 18 shows a 1200 amp current pulse and the corresponding forces produced. At 1200 amps, the Lorentz force magnitude is approximately 10 times greater than the peak recoil reading, and 30 times greater than the steady-state recoil. The data shows that higher currents resulted in recoil forces of the same magnitude, as seen in Figure 19. These recoil readings are interpreted as artifacts of the experiment. For larger currents, the I^2R losses produced enough heat to raise the copper's resistance. This created the declining current levels seen in many current pulses.

Additional artifacts are labeled in Figure 18 for explanation. Table vibrations occurred whenever the high current switch was opened or closed. The load cells detected all vibrations, since they were adjusted to be in contact, or preloaded. When preloaded, the force reading was set to zero via Labview. Upon separation, this caused the load cell to produce a negative force reading, or preload release. To measure a force, the load cell had to be in contact with the rail support or armature. This contact between the stainless steel load cell and hard plastic created bouncing if the two separated and came back together. These bounces appeared as force oscillations on the graphs.

As current began to flow, there were transient mechanical oscillations in the rails and eutectic, which

caused the recoil peak. While current flowed through the eutectic in the armature, the liquid metal was pushed forward by the Lorentz force. When current stopped flowing, the eutectic flowed back and the armature would swing back and bump the rails. This caused the large peaks in recoil after current flow had stopped.
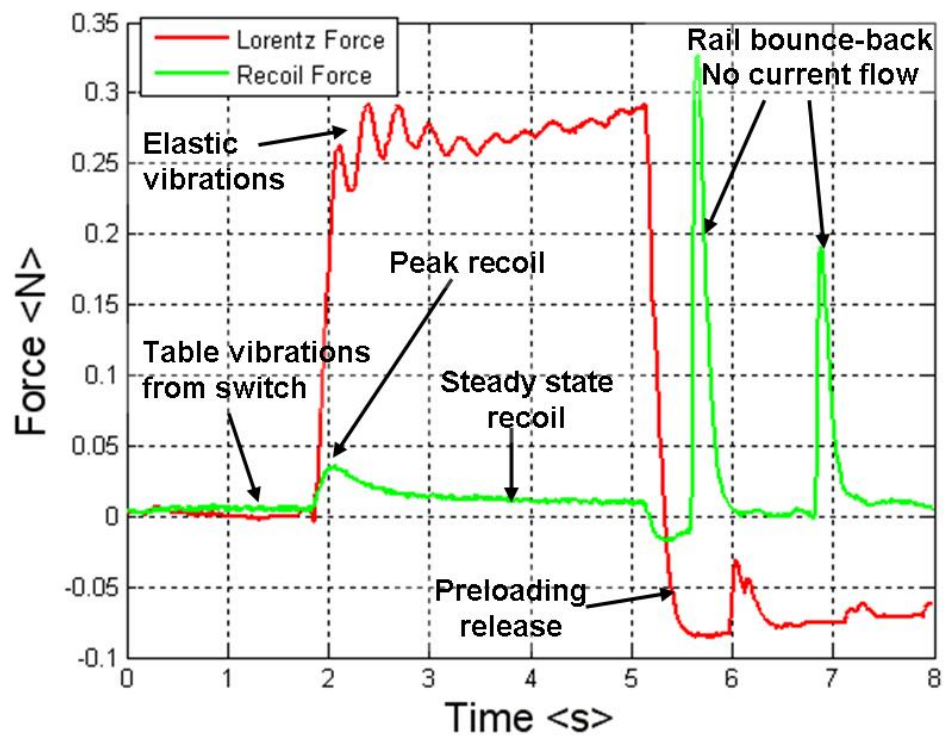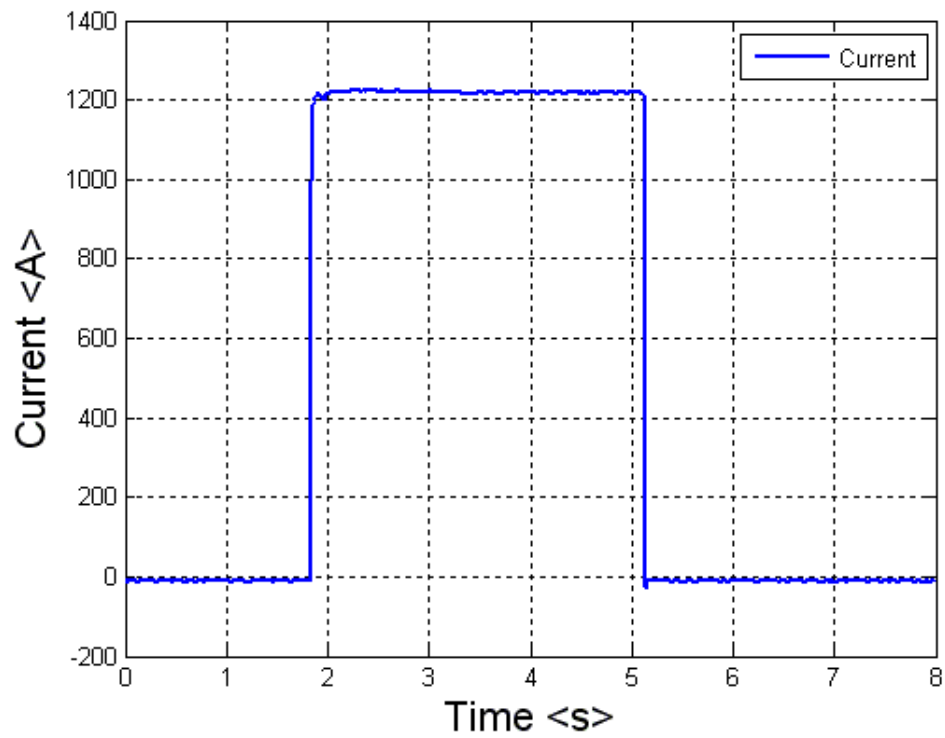
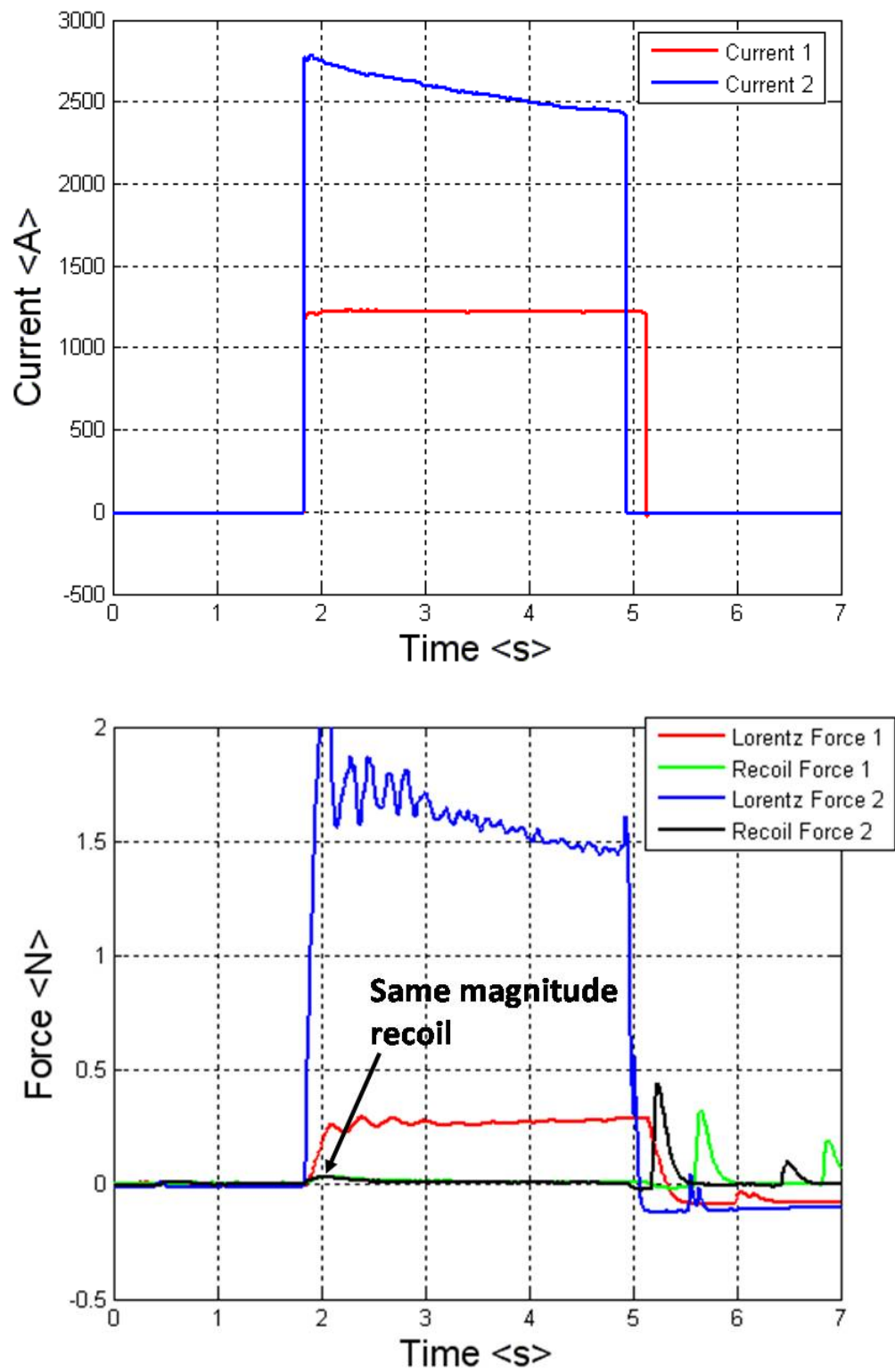Figure 18. Armature Lorentz force and rail recoil force
for 1.2 kA pulse

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Figure 19. Armature Lorentz forces and rail recoil forces
for 1.2 kA and 2.5 kA pulses

Tables 1 and 2 show how little the peak and steady-state rail recoil changed, regardless of the current and armature Lorentz force readings. The Lorentz force column refers to the force on the armature.

Current (A)	Lorentz Force (N)	Recoil Force-Peak (N)
857.8	0.18	0.036
1218.0	0.29	0.036
1539.3	0.443	0.036
1715.6	0.60	0.024
1849.7	0.72	0.034
2604.9	2.24	0.038
2683.8	2.64	0.030
2741.7	2.70	0.032

Table 1.   Peak rail recoil force measurements

Current (A)	Lorentz Force (N)	Recoil Force-Steady State (N)
863.7	0.17	0.0039
1218.0	0.29	0.0098
1503.8	0.46	0.0106
1681.0	0.59	0.0063
1779.4	0.66	0.0086
2017.3	0.92	0.0083
2235.1	1.23	0.0078
2373.5	1.40	0.0119
2602.7	1.73	0.0102

Table 2.   Steady-state rail recoil forces

The data in Table 2 is plotted in Figure 20. The Lorentz force on the armature is directly proportional to the square of the current while the recoil doesn't show consistent or predictable current dependence. The measured armature force is consistent with Equation (1). The steady-state force for each data point measures approximately 0.01 N, or less. The complete real-time graph for each data point can be viewed in Appendix section 1.
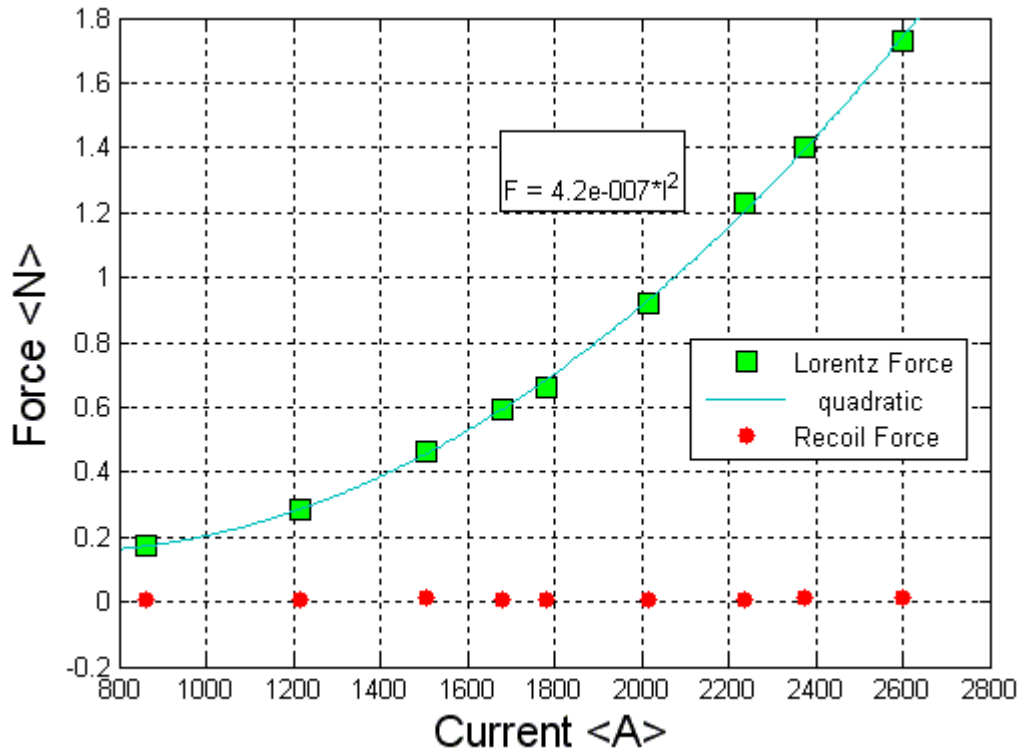

Figure 20. Armature Lorentz force and steady state rail recoil vs. current

B. SPLIT RAIL MEASUREMENTS

1. Armature Lorentz Force and Recoil on the Muzzle Half Rail

Results of the force comparison between armature Lorentz force and rail recoil did not change for the muzzle half of the rails once they were split. Figure 21 shows a nearly 2 kA current pulse and the forces measured. Results for other current levels can be view in Appendix section 2. During brief transient oscillations, as the circuit energized, the recoil peaked at 0.05 N, which is approximately 2% of the magnitude of the armature Lorentz force produced. After the transient, the steady-state recoil measured less than 0.01 N, which is less than 1% of the armature Lorentz force measured.
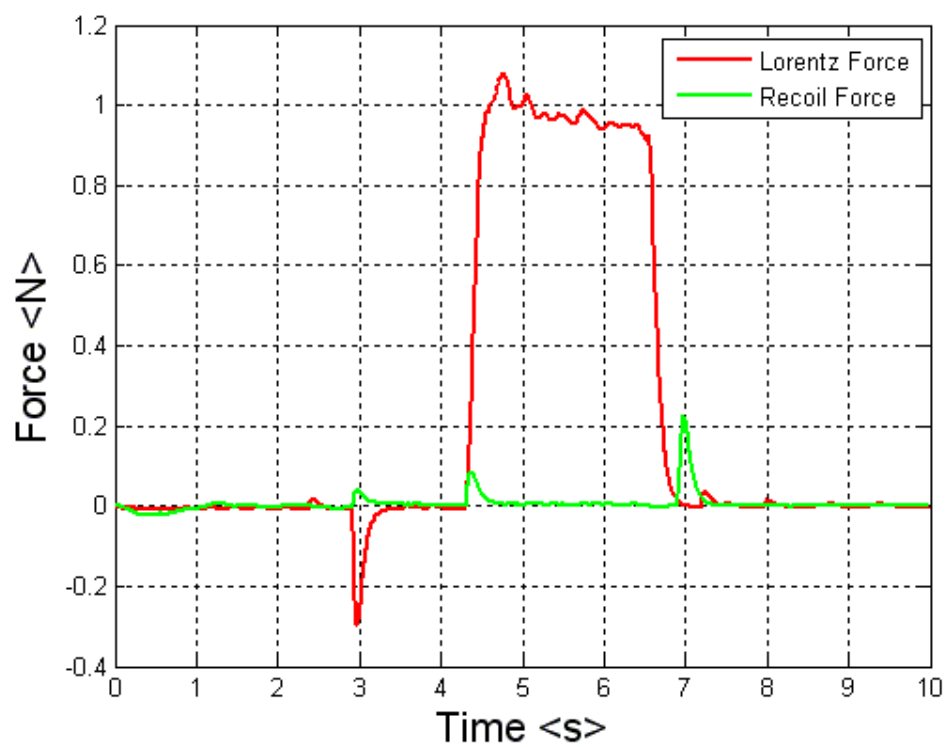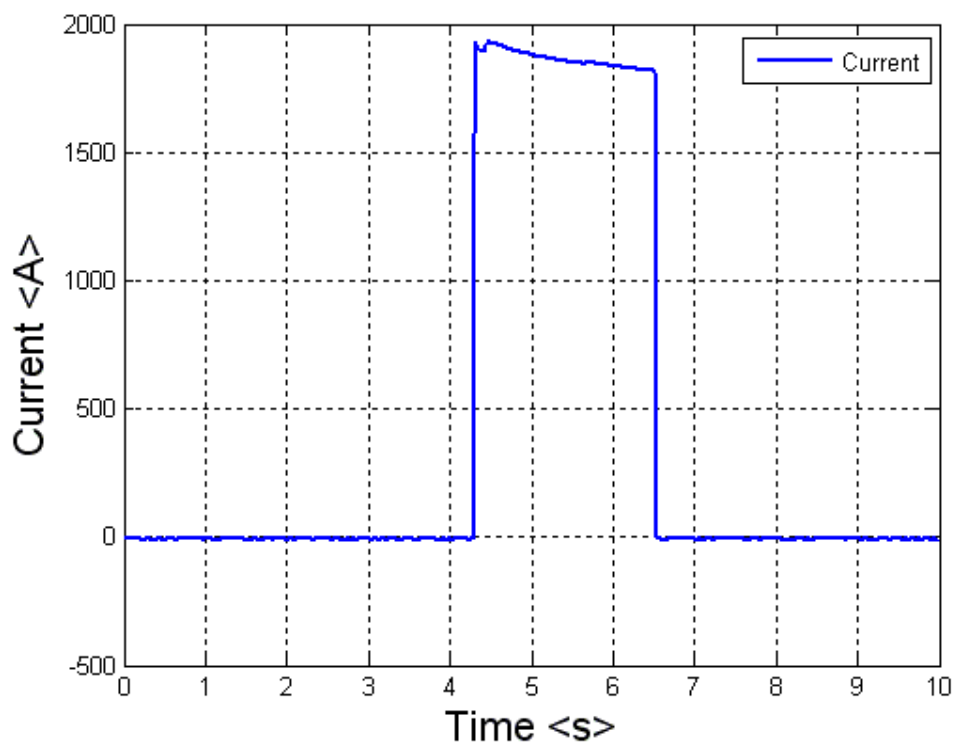
Figure 21.   Split rails-armature Lorentz force and rail
recoil force for 1.8 kA pulse

2.   Forces Between the Rail Halves

Before the rails were split, it was evident that there was no net force on the rails. Splitting the rails was necessary to determine if there were equal and opposite forces being exerted on the rails. If recoil were seated in the rails and a canceling force from the breach region existed, then the rail halves would push together toward the center of the rails. Instead, the rails were found to push apart slightly. Figure 22 shows a 2 kA current pulse and equal and opposite forces of approximately 0.22 N being exerted on the rails. From separate measurements, similar current pulses created a Lorentz force of approximately 1 N. With the creation of two more sets of tabs dipped into eutectic, undesired vertical current components were introduced and the interactions of these with magnetic fields would exert forces separating the two halves of the rails. If one-dimensional current could have been achieved while splitting the rails, the author believes these opposing forces would not have existed. Furthermore, these equal and opposite forces are not believed to exist as internal stress within unbroken rails.
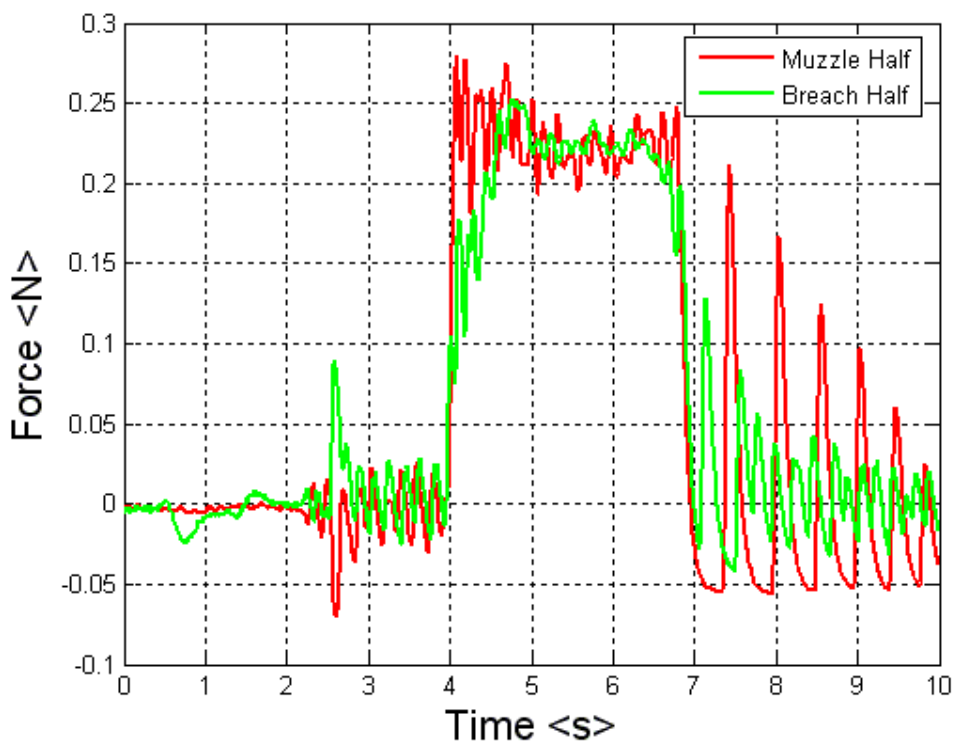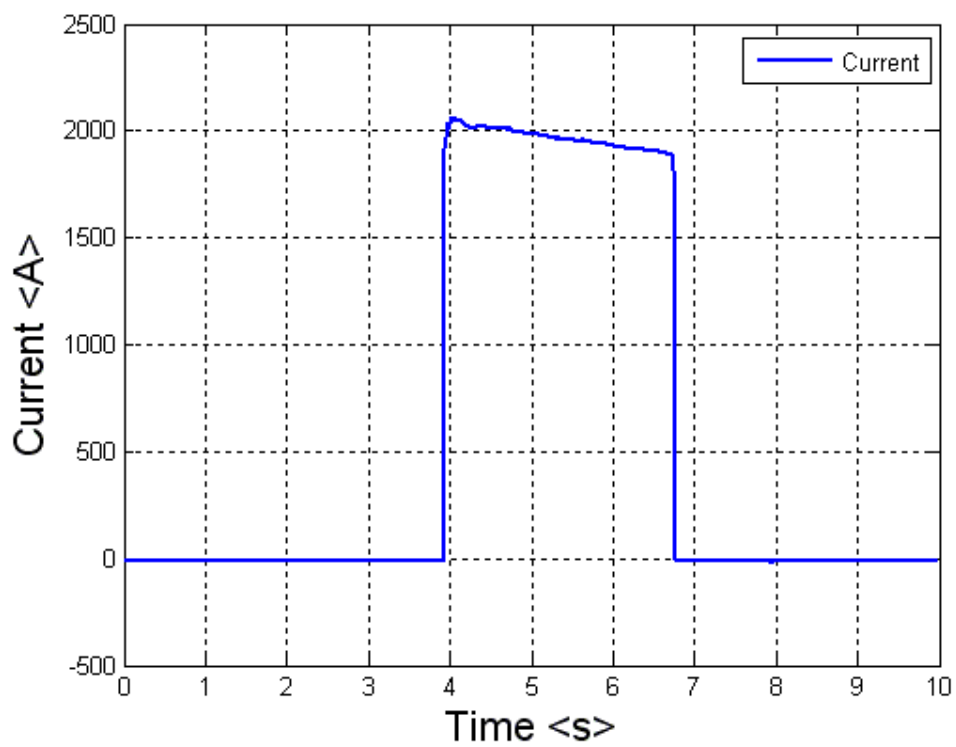
Figure 22. Split rail — Opposing rail forces for
2 kA current pulse

C.   SUMMARY

An efficient and effective electromagnetic railgun design rests on a thorough understanding of the forces at work within the gun. The research in this thesis addresses the controversial question, "Are recoil forces seated in the rails?" This question impacts how the gun should be designed, and what resources would be needed. The weight, size, and durability will be among the primary concerns when EM railguns are installed on ships.

This experiment investigated recoil exerted on the rails by simultaneously measuring armature Lorentz force and rail recoil with real-time data recording. If the recoil was seated in the rails, it was expected to have a magnitude nearly equal to the Lorentz force in the opposite direction. Simultaneous measurements over a large range of currents were compared. The max current attained was 2.7 kA, and the measured Lorentz force was 1.7 N, while the recoil peaked at less than 2% of this value and then dropped to less than 1%, as seen in Figure 19. Appendix section 1 shows graphical results for various current levels, which are consistent with the results in Figure 19. The recoil readings are not current dependent, and are interpreted as artifacts of the experiment.

Splitting the rails and simultaneously measuring armature Lorentz force and recoil on the muzzle half of the rails yielded results consistent with those for the unsplit rails. The maximum current attained with this setup was 1.9 kA, and the measured Lorentz force was approximately 1 N, while the steady-state recoil was less than 1% of this value, as seen in Figure 21. The equal and opposite forces

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pushing the split rails apart in Figure 22 are interpreted as an artifact of the experiment, and are not associated with recoil in any way. The fact that the split rails did not push toward each other, combined with the results from the split rail Lorentz-recoil measurements (Figure 21), leads to the conclusion that there are not any internal stresses within the rails.

Since there are no indications of internal stresses and the simultaneous Lorentz-recoil measurements do not indicate a Lorentz reaction force on the rails, this experimental investigation has shown that recoil forces are not seated in the rails.

APPENDIX

1. LORENTZ FORCE AND RECOIL
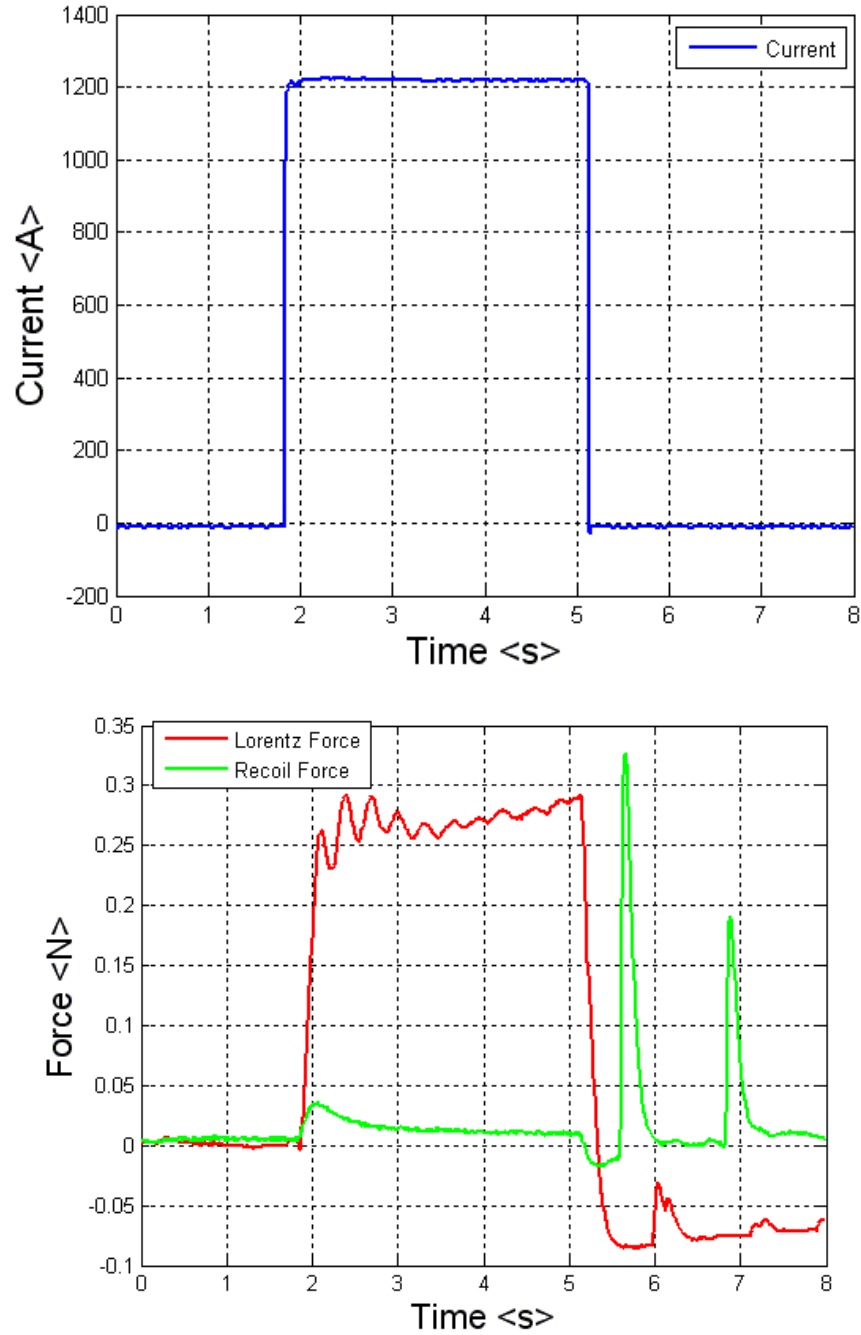
Figure 23.   Armature Lorentz force and rail recoil force for
1.2 kA pulse
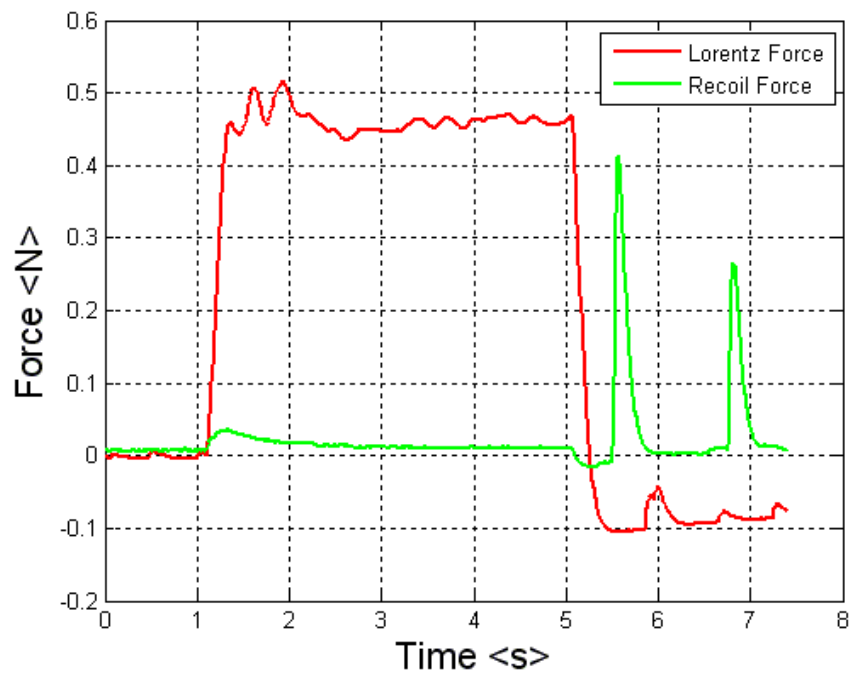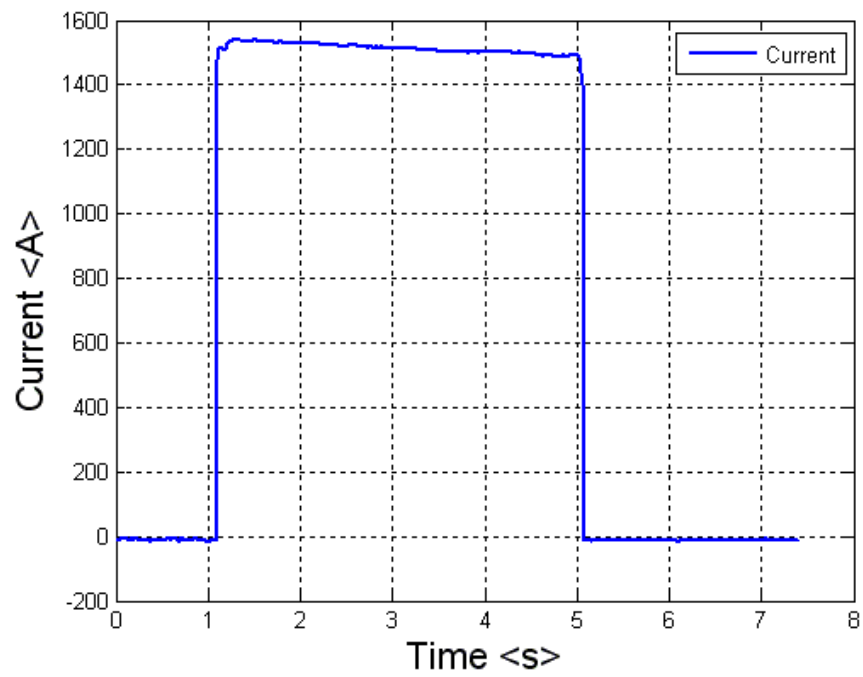
Figure 24.   Armature Lorentz force and rail recoil force for
1.5 kA pulse
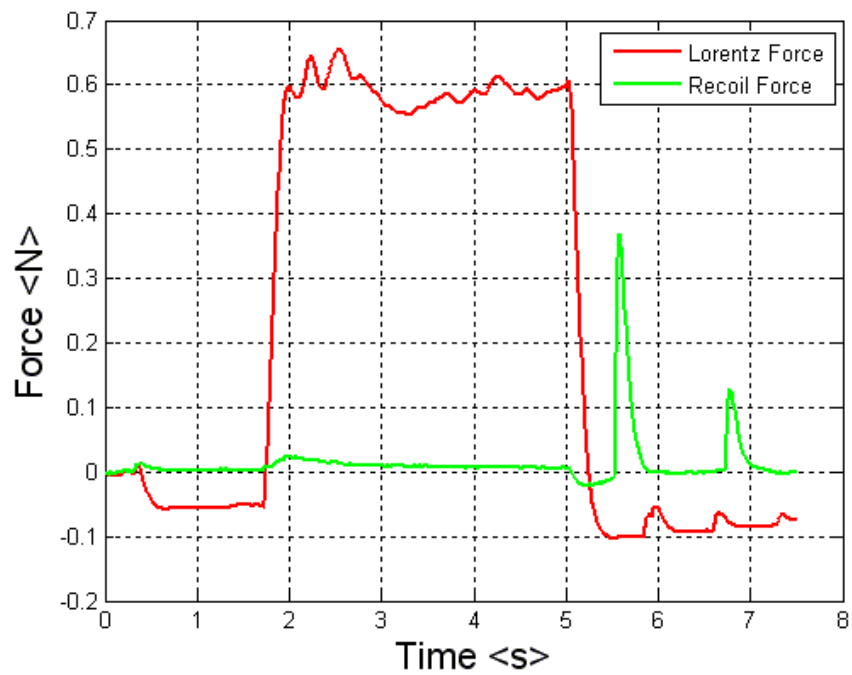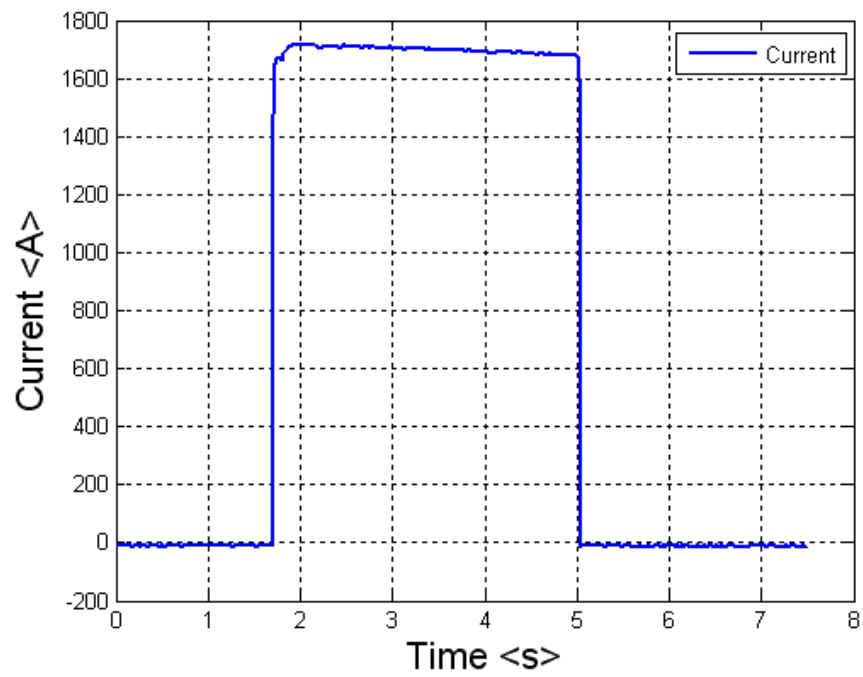
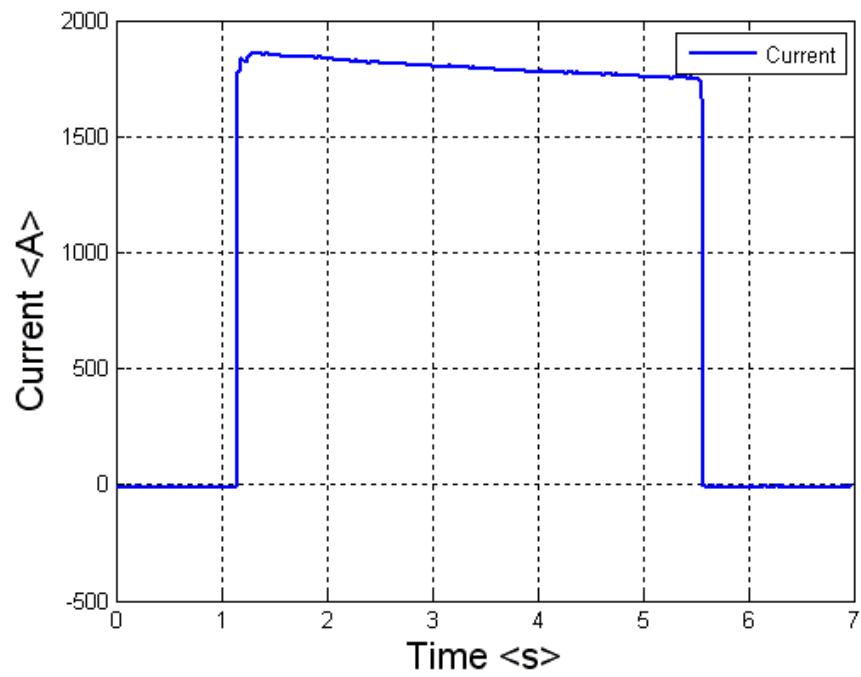Figure 25.   Armature Lorentz force and rail recoil force for
1.7 kA pulse
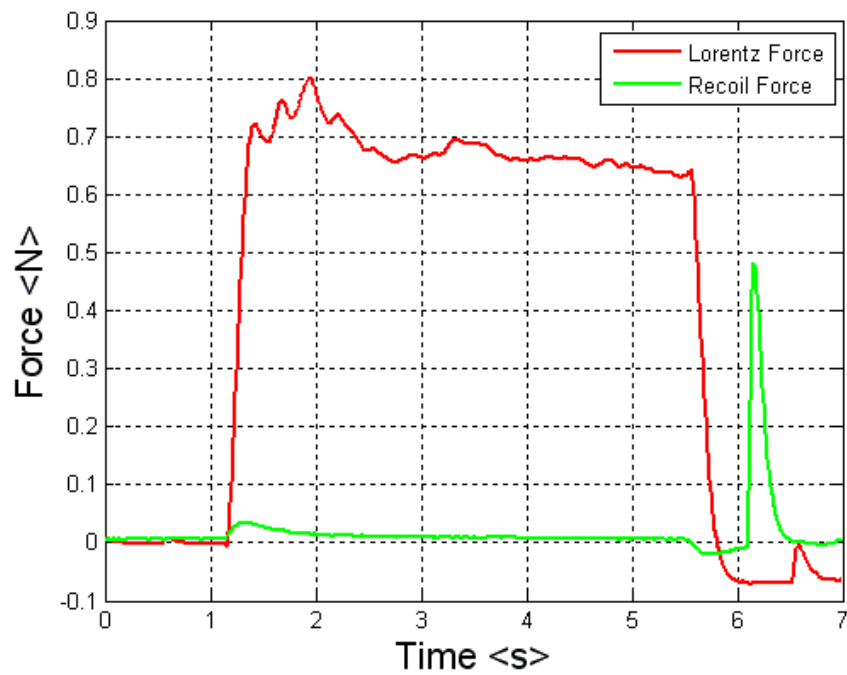
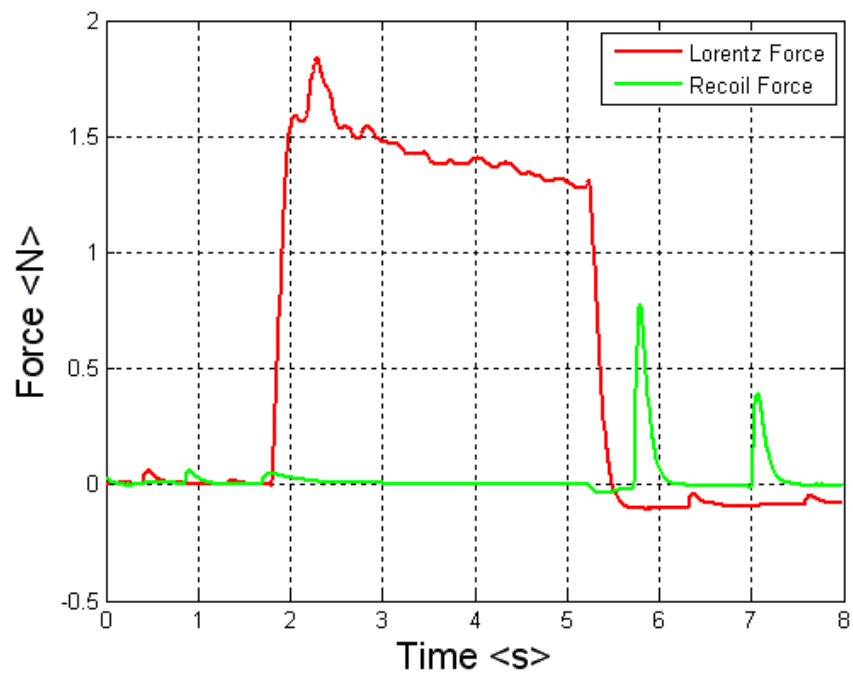Figure 26.   Armature Lorentz force and rail recoil force for
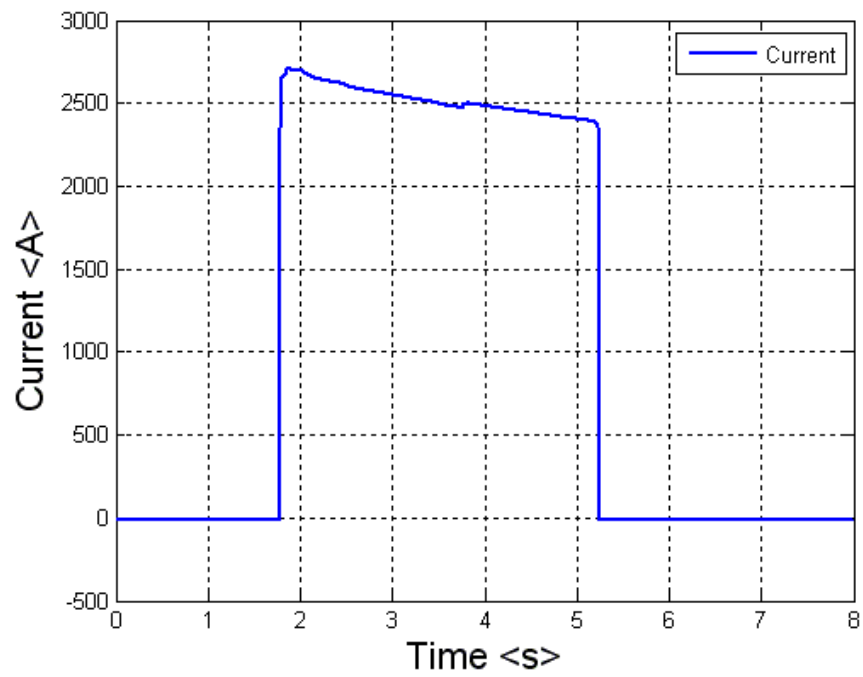1.8 kA pulse

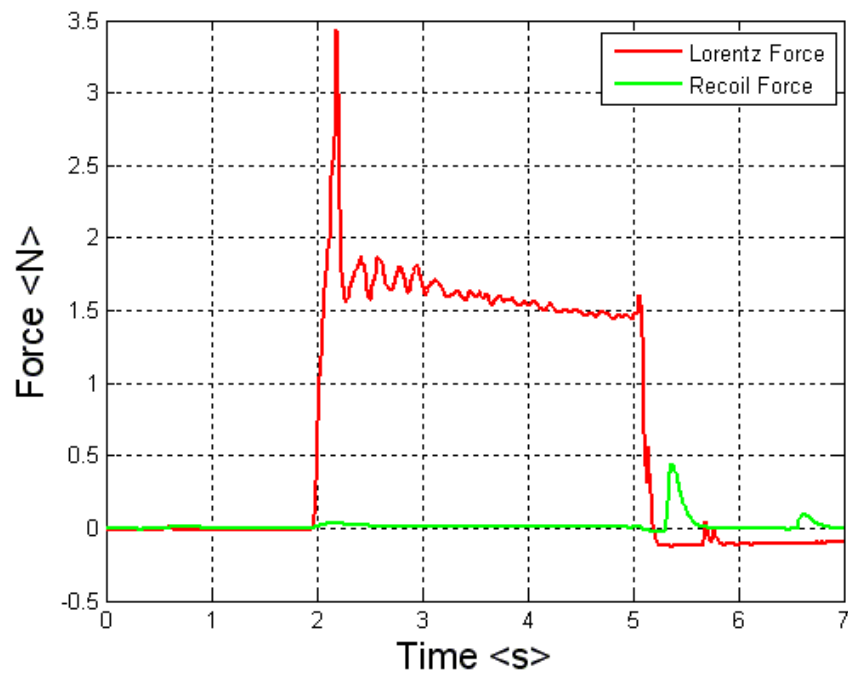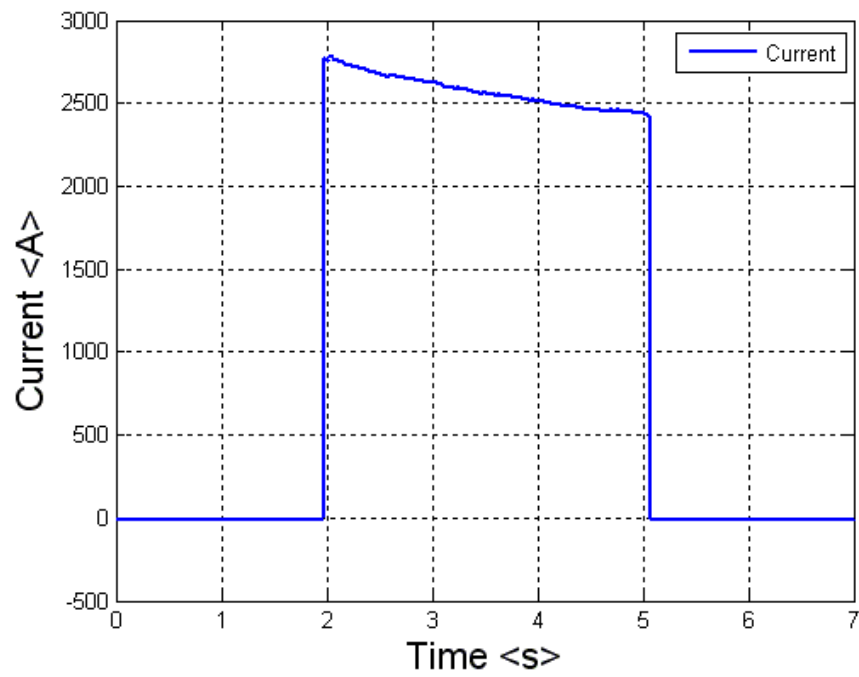Figure 27.   Armature Lorentz force and rail recoil force for
2.6 kA pulse

Figure 28.   Armature Lorentz force and rail recoil force for
2.7 kA pulse

2.   SPLIT RAILS—LORENTZ FORCE AND MUZZLE HALF RECOIL
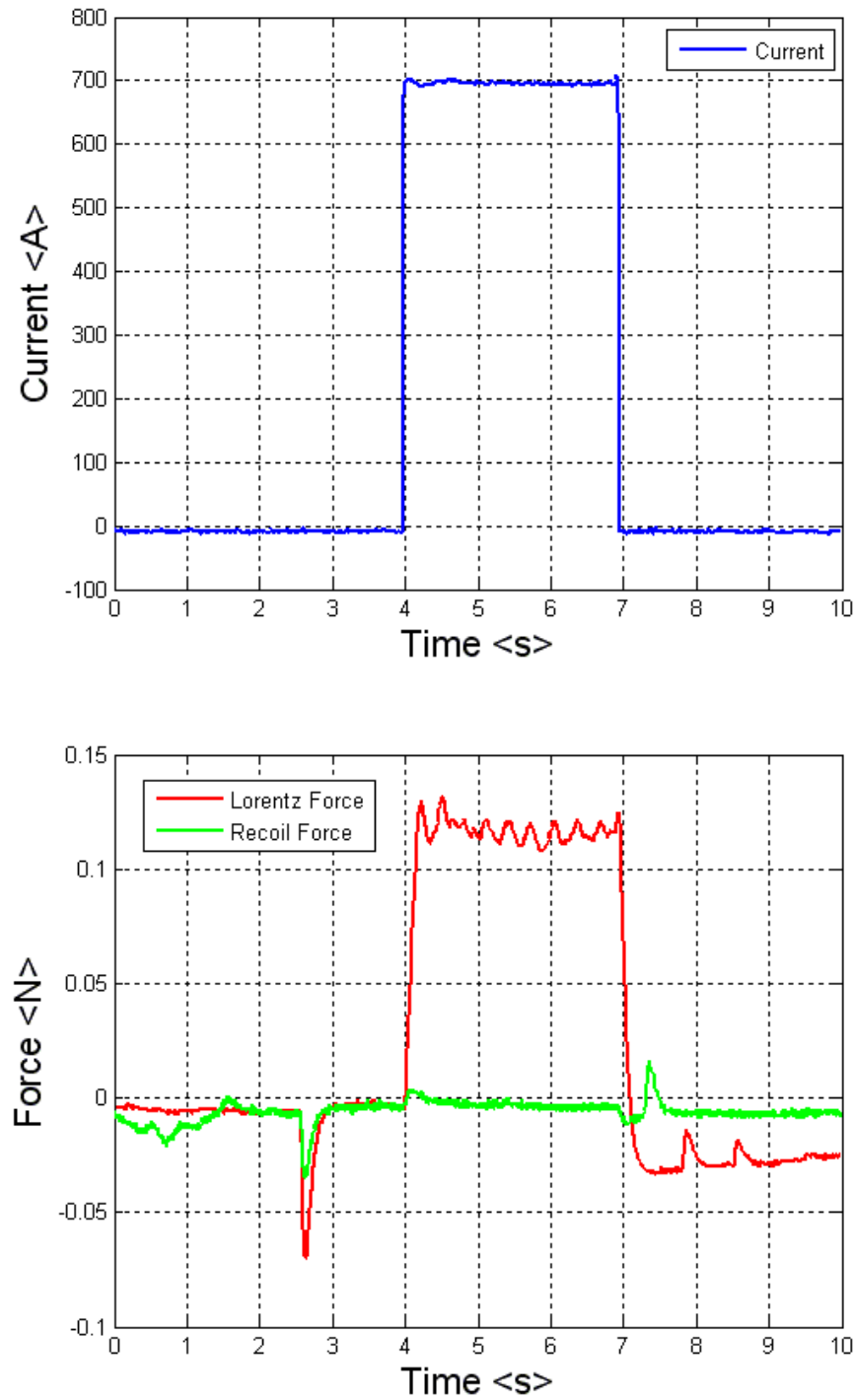

Figure 29.   Split rail — armature Lorentz force and recoil
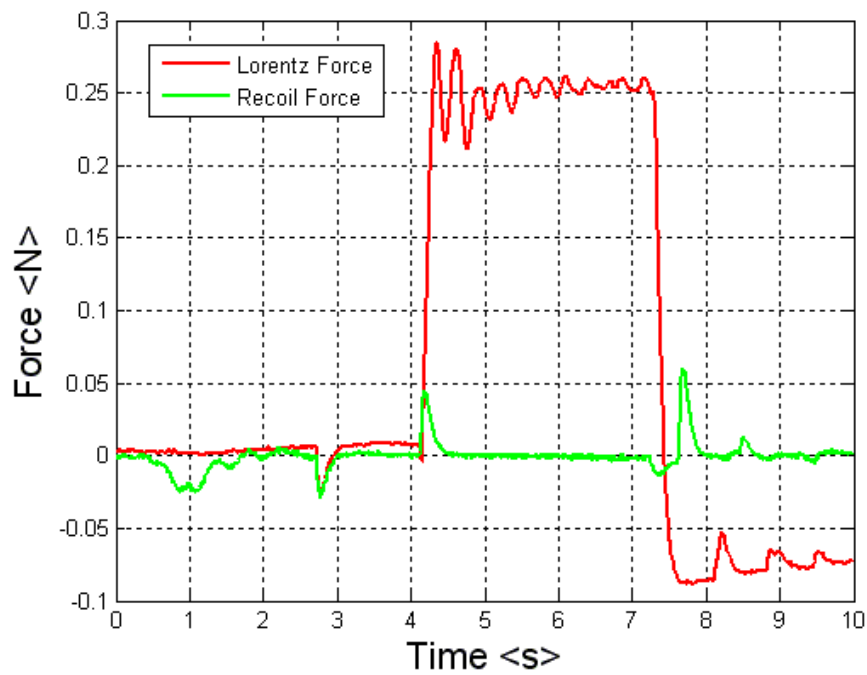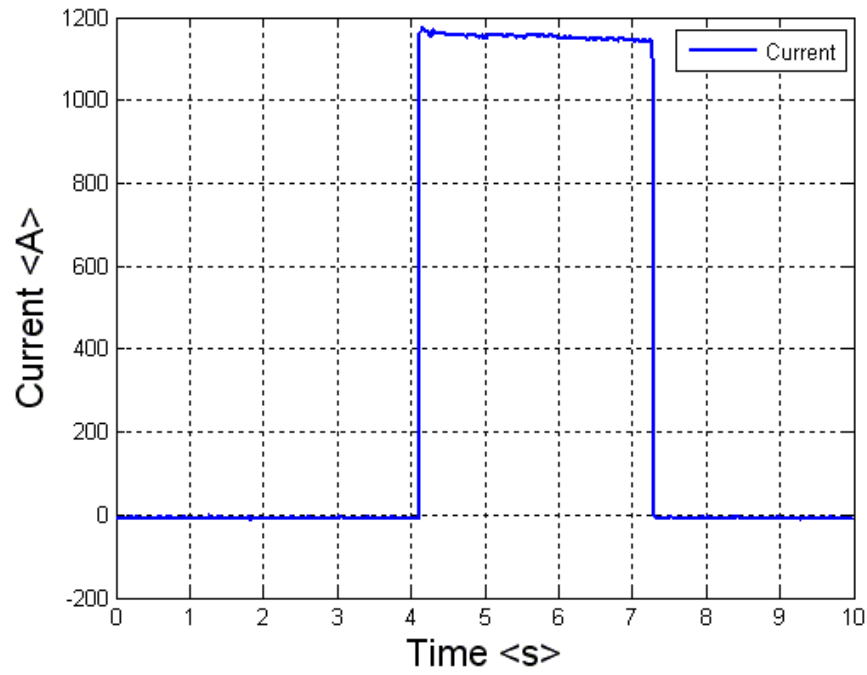force for 0.7 kA pulse

Figure 30.   Split rail — armature Lorentz force and recoil
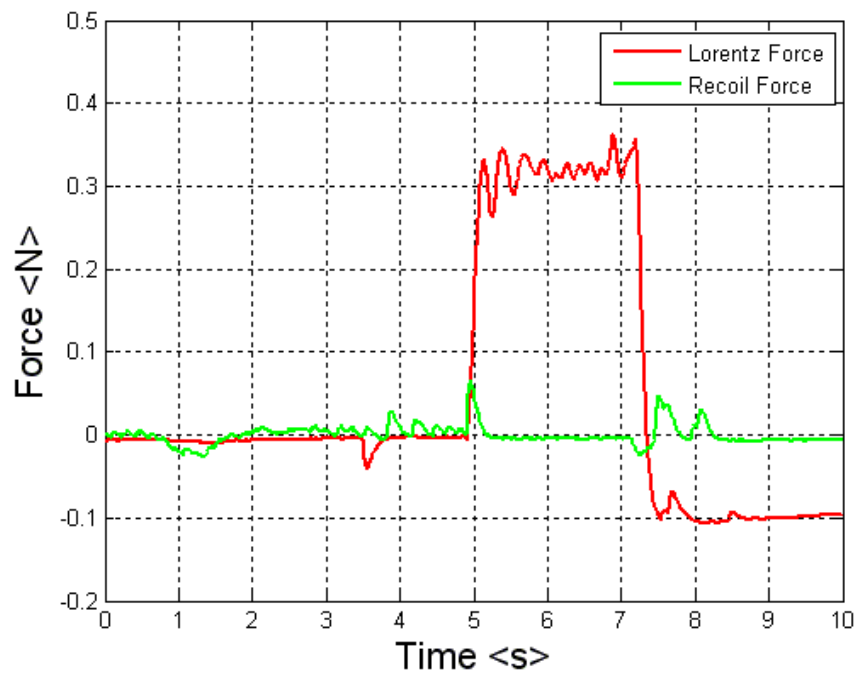force for 1.2 kA pulse

Figure 31.   Split rail − armature Lorentz force and recoil
force for 1.3 kA pulse

Figure 32.   Split rail — armature Lorentz force and recoil
force for 1.3 kA pulse
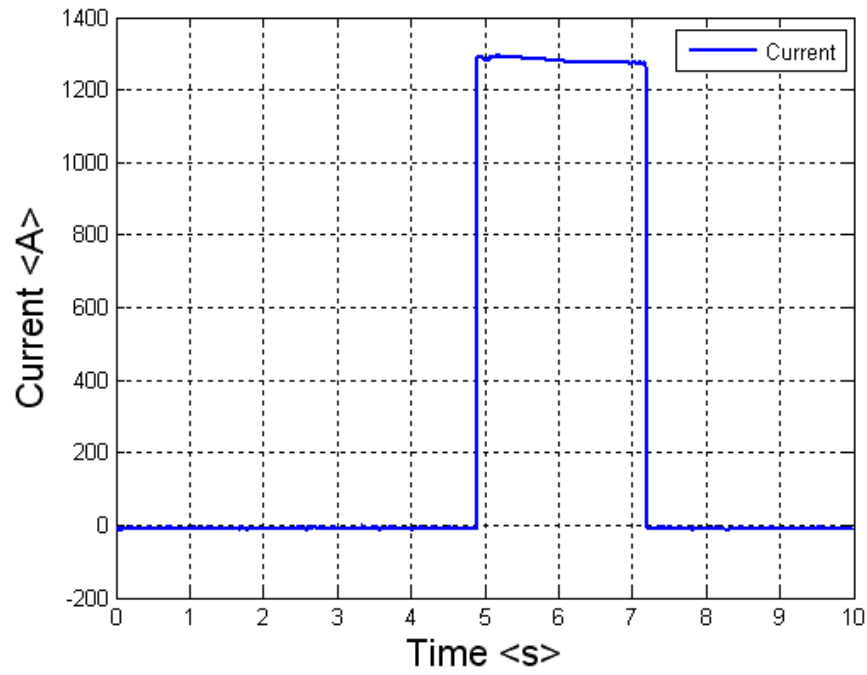
Figure 33.   Split rail — armature Lorentz force and recoil
force for 1.4 kA pulse
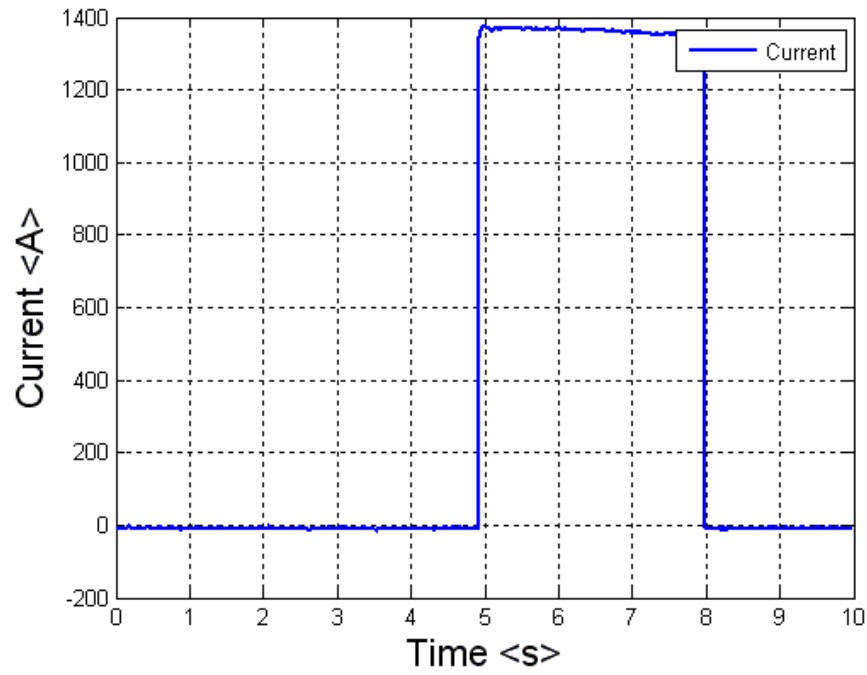
Figure 34.   Split rail — armature Lorentz force and recoil
force for 1.4 kA pulse
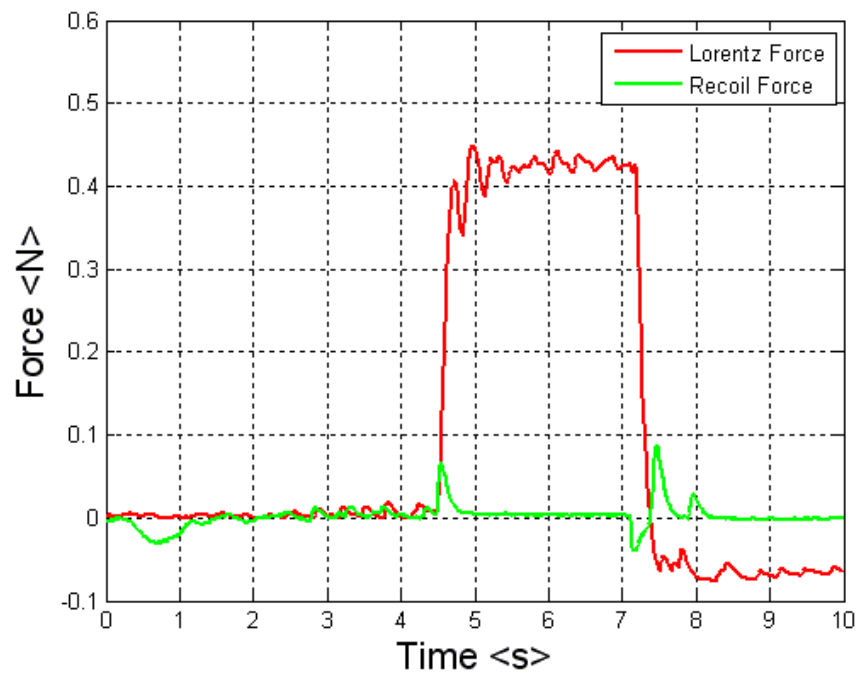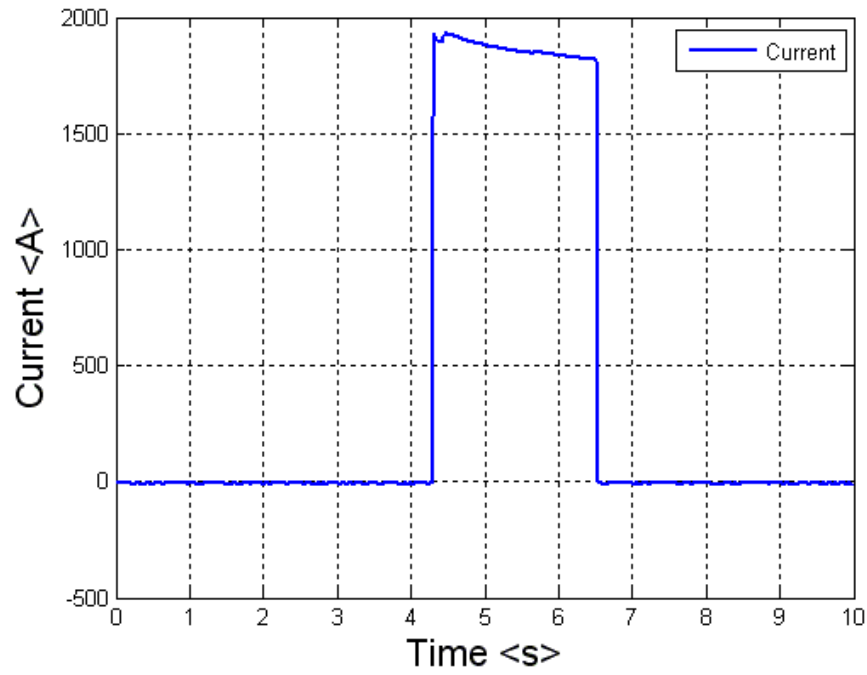
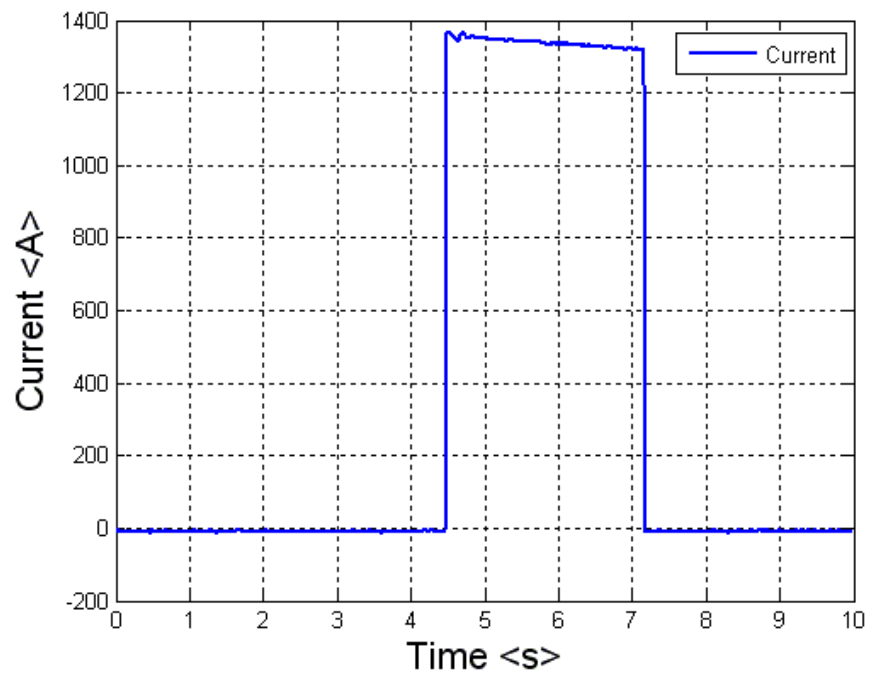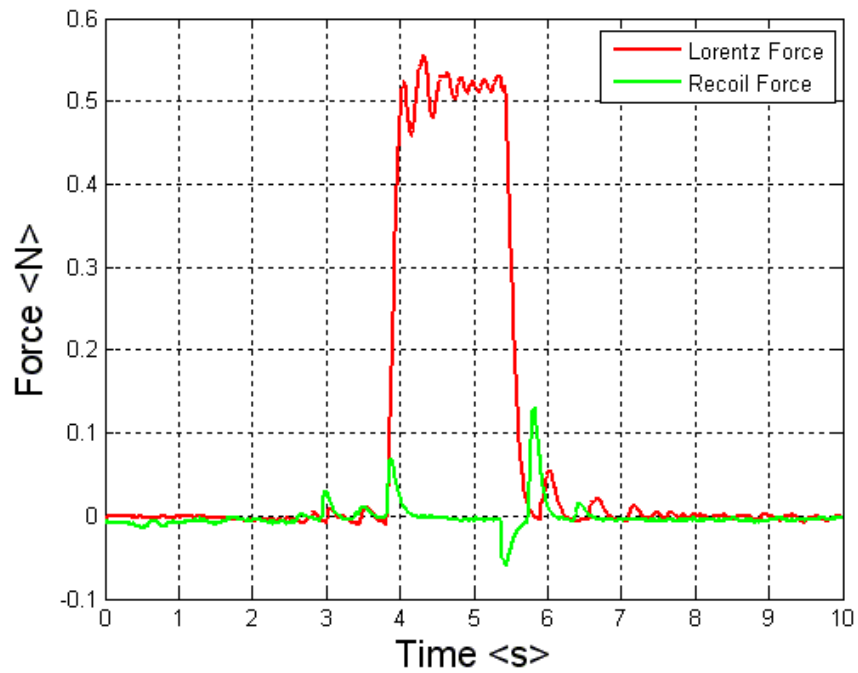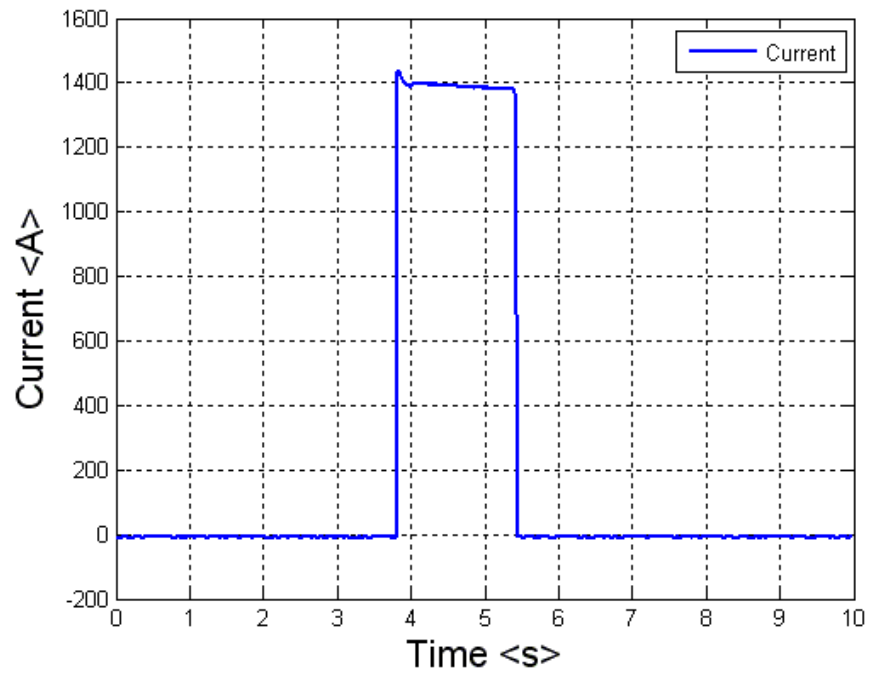Figure 35.   Split rail − armature Lorentz force and recoil
force for 1.9 kA pulse
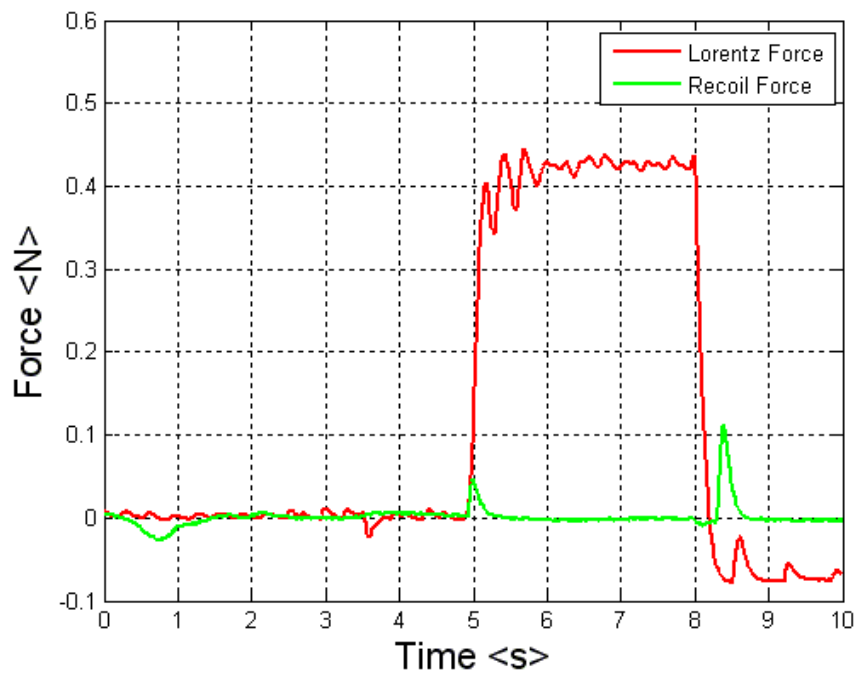
Figure 36.   Split rail — armature Lorentz force and recoil
force for 2 kA pulse

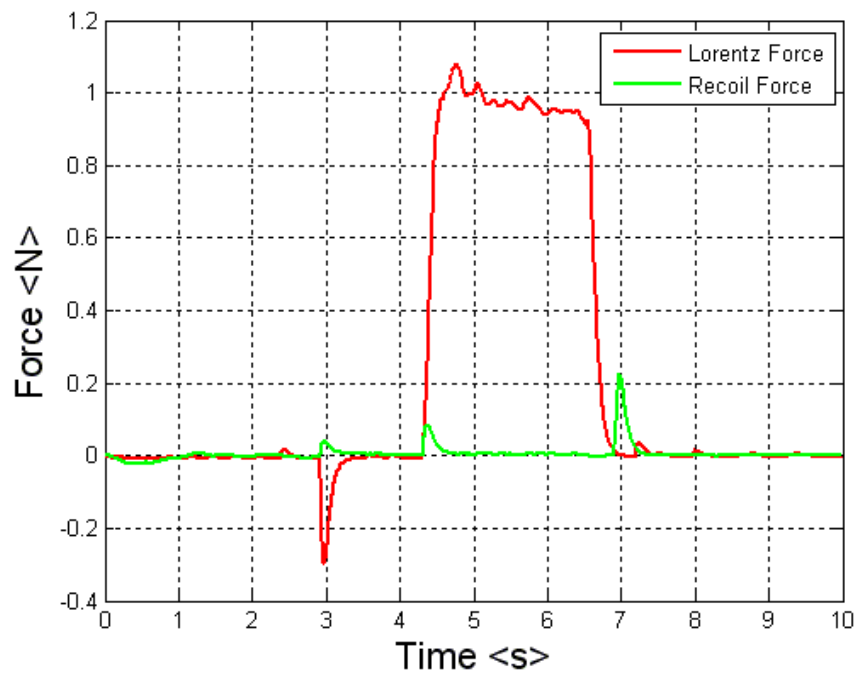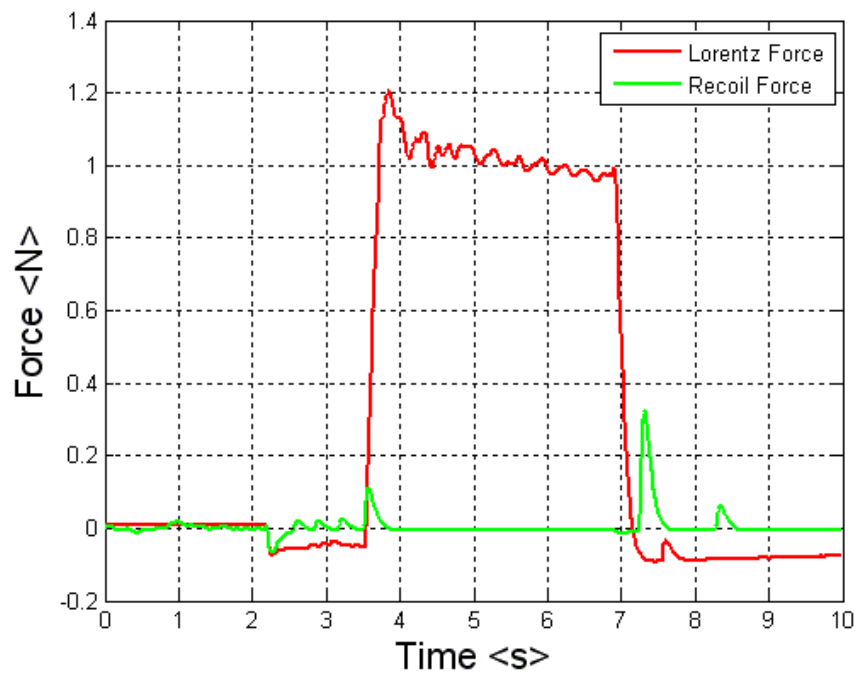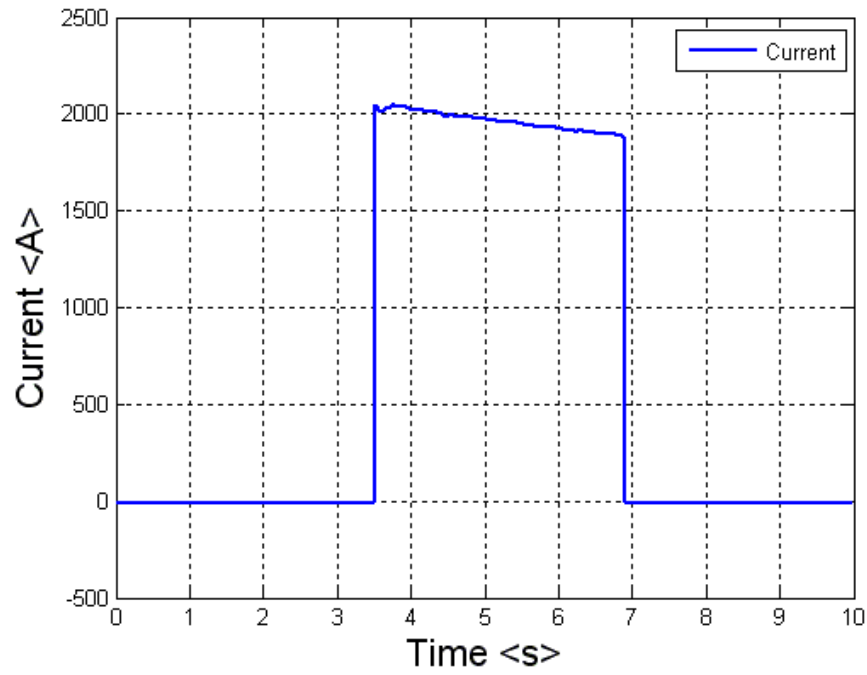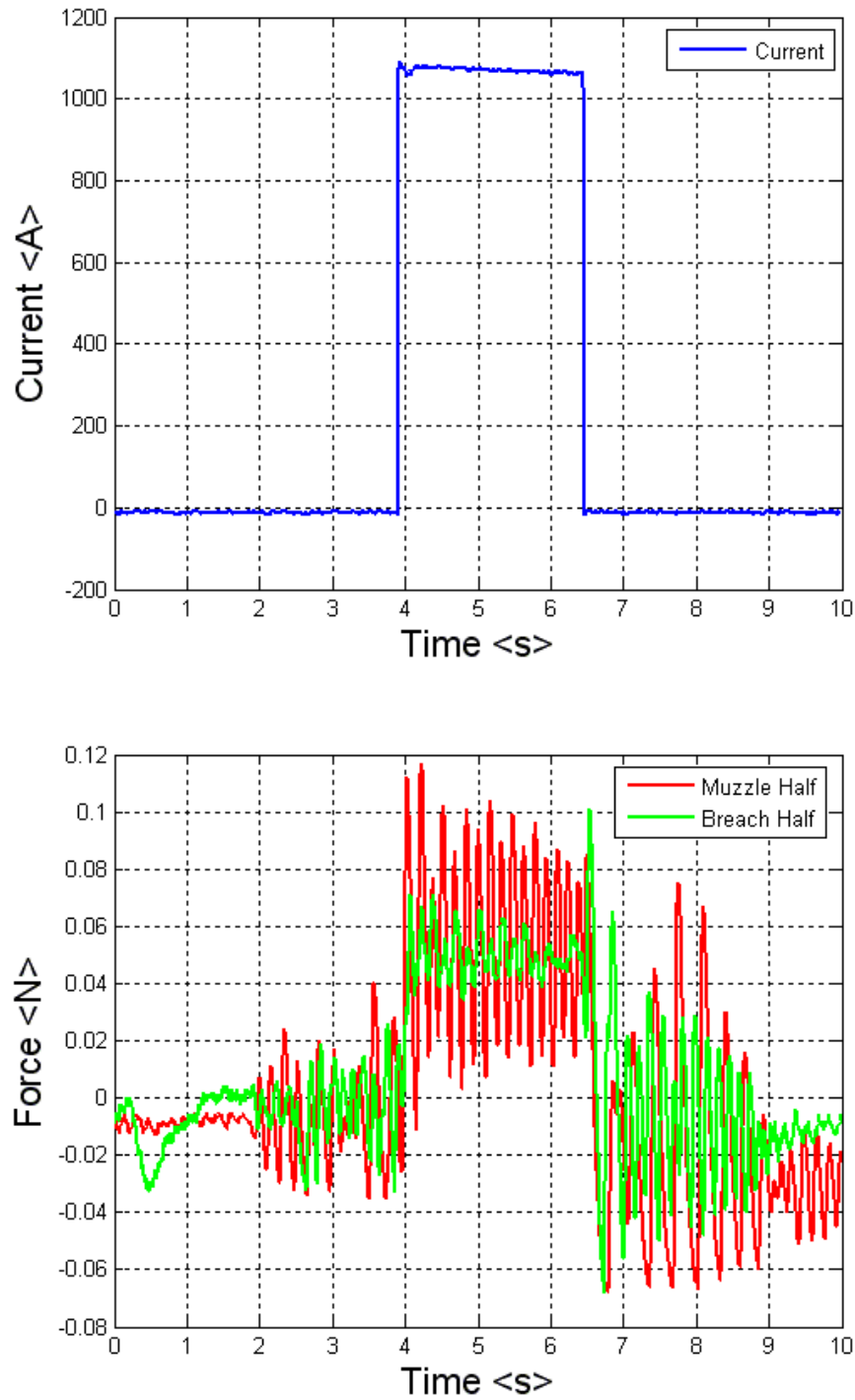**3.   SPLIT RAILS—OPPOSING FORCES ON EACH RAIL HALF**

Figure 37.   Split rail — opposing rail forces for
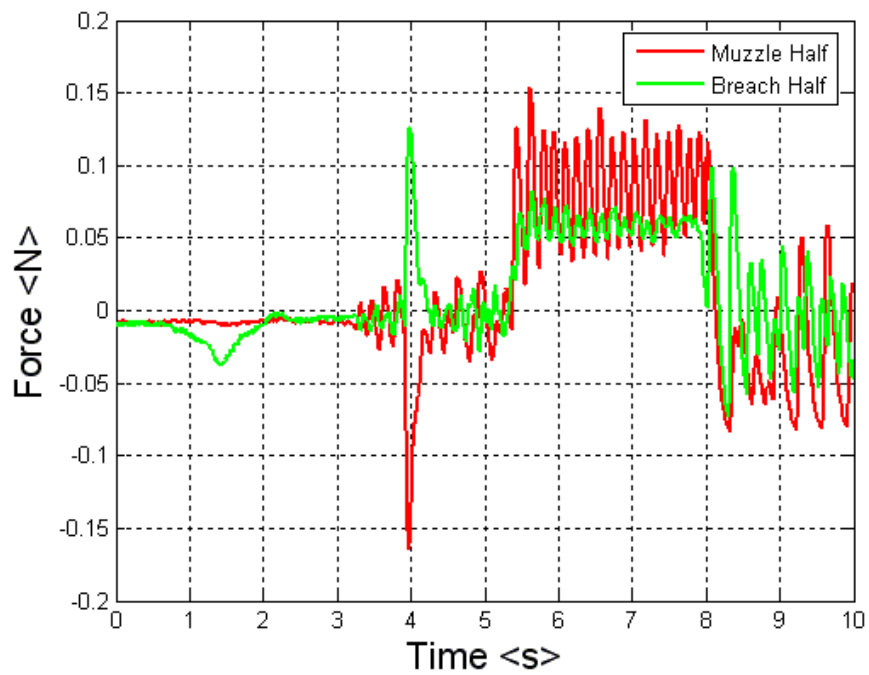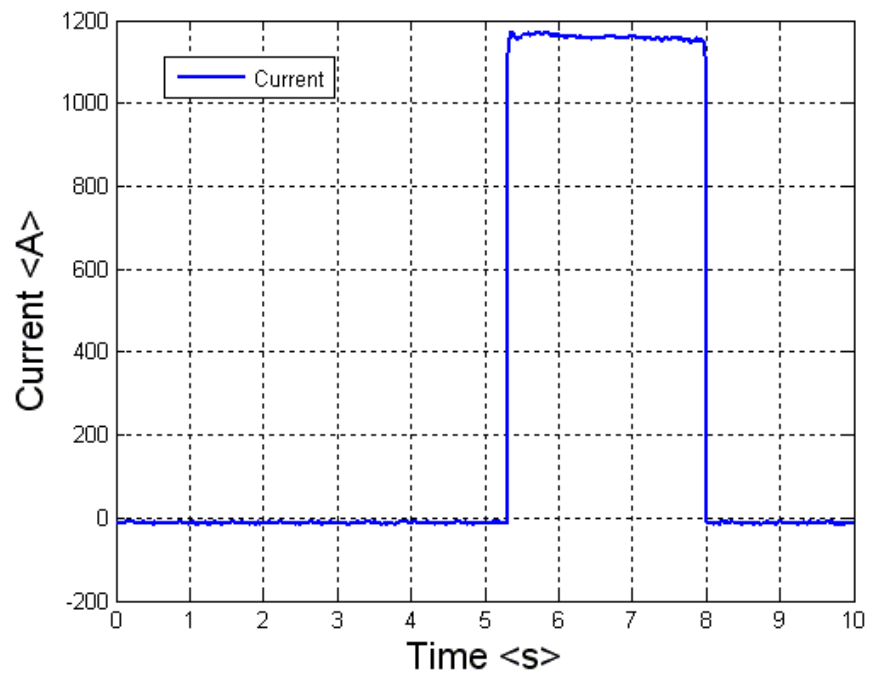1.1 kA current pulse

Figure 38.   Split rail — opposing rail forces for
1.2 kA current pulse
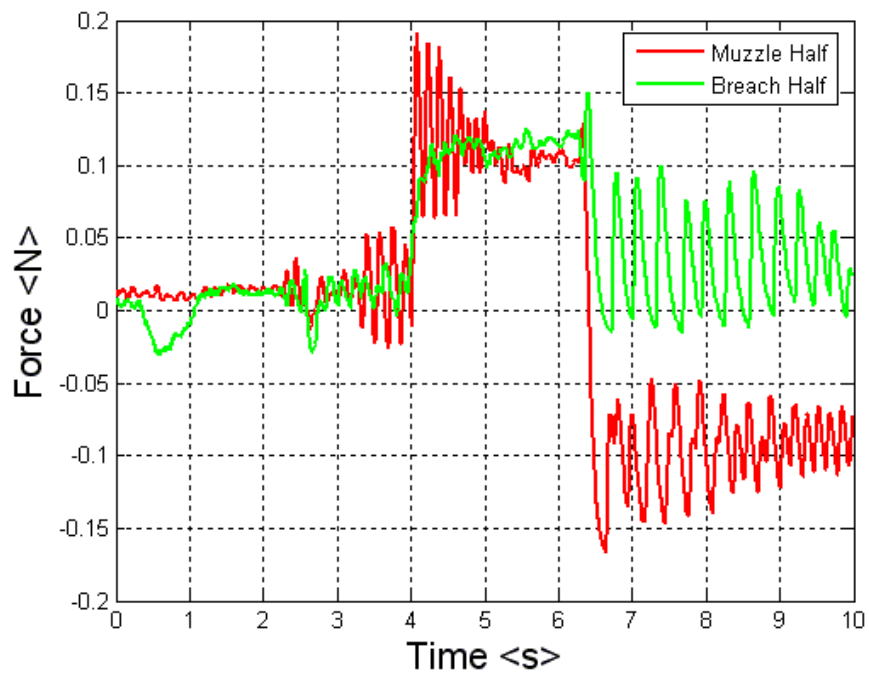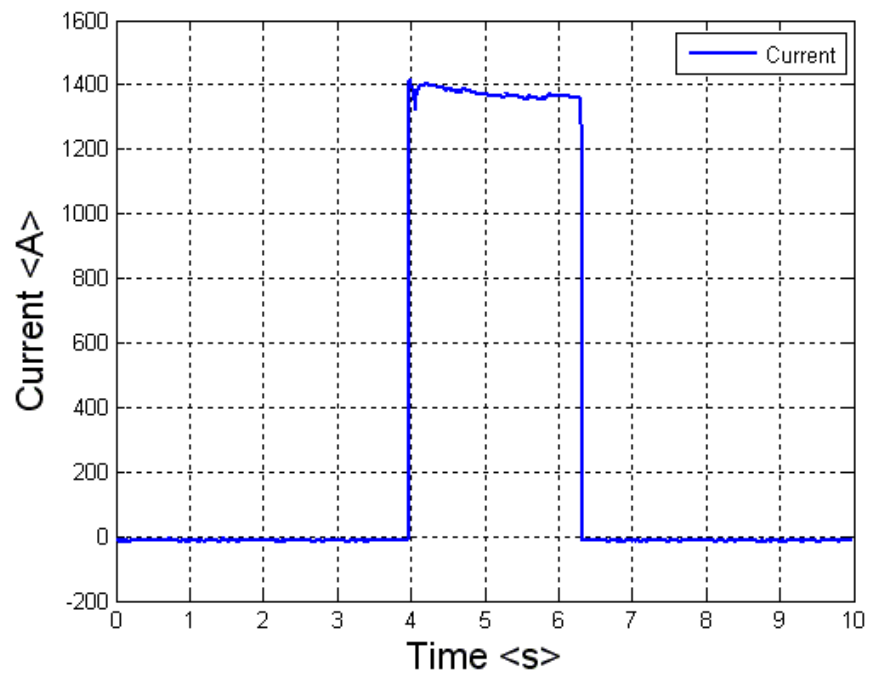
Figure 39.   Split rail — opposing rail forces for
1.4 kA current pulse

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